Terminology: Range Notation

- m..n is a range containing n+1-m values
  - 2..5 contains 2, 3, 4, 5. Contains 5+1 – 2 = 4 values
  - 2..4 contains 2, 3, 4. Contains 4+1 – 2 = 3 values
  - 2..3 contains 2, 3. Contains 3+1 – 2 = 2 values
  - 2..2 contains 2. Contains 2+1 – 2 = 1 values
  - 2..1 contains ???

What does 2..1 contain?

A: nothing
B: 2,1
C: 1
D: 2
E: something else

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Assertions: Tracking Code State

- **assertion**: true-false statement placed in a program to *assert* that it is true at that point
  - Can either be a **comment**, or an **assert** command
- **invariant**: assertion supposed to "always" be true
  - If temporarily invalidated, must make it true again
  - **Example**: class invariants and class methods
- **loop invariant**: assertion supposed to be true before and after each iteration of the loop
- **iteration of a loop**: one execution of its body

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Assertions versus Asserts

- Assertions **prevent bugs**
  - Help you keep track of what you are doing
- Also **track down bugs**
  - Make it easier to check belief/code mismatches
- The assert statement is a (type of) assertion
  - One you are **enforcing**
  - Cannot always convert a comment to an assert

```
# x is the sum of 1..n
```

Comment form of the assertion.

The root of all bugs!

x	?	n	1
x	?	n	3
x	?	n	0

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Preconditions & Postconditions

precondition

```
# x  = sum of 1..n-1
x = x + n
n = n + 1
# x =  sum of 1..n-1
```

postcondition

- **Precondition:** assertion placed before a segment
- **Postcondition:** assertion placed after a segment

n
1 2 3 4 5 6 7 8
x contains the sum of these (6)

n
1 2 3 4 5 6 7 8
x contains the sum of these (10)

Relationship Between Two
If precondition is true, then postcondition will be true

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Solving a Problem

precondition

```
# x  = sum of 1..n

n = n + 1
# x =  sum of 1..n
```

postcondition

What statement do you put here to make the postcondition true?

A: x = x + 1
B: x = x + n
C: x = x + n+1
D: None of the above
E: I don't know

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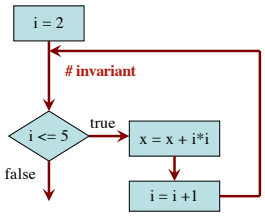
Invariants: Assertions That Do Not Change

- **Loop Invariant**: an assertion that is true before and after each iteration (execution of repetend)

```
x = 0; i = 2
while i <= 5:
    x = x + i*i
    i = i +1
# x = sum of squares of 2..5
```

Invariant:
x = sum of squares of 2..i-1

in terms of the range of integers that have been processed so far



The loop processes the range 2..5

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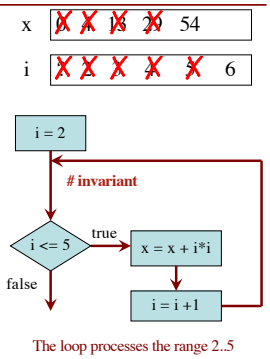
Invariants: Assertions That Do Not Change

```
x = 0; i = 2
# Inv: x = sum of squares of 2..i-1
while i <= 5:
    x = x + i*i
    i = i +1
# Post: x = sum of squares of 2..5
```

Integers that have been processed: 2, 3, 4, 5

Range 2..i-1: 2..5

Invariant was always true just before test of loop condition. So it's true when loop terminates



The loop processes the range 2..5

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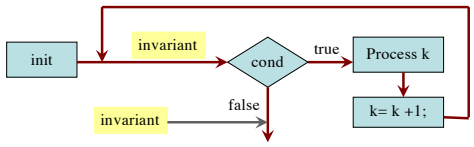
Designing Integer while-loops

```
# Process integers in a..b
# inv: integers in a..k-1 have been processed
k = a
while k <= b:
    process integer k
    k = k + 1
# post: integers in a..b have been processed
```

Command to do something

Equivalent postcondition



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Designing Integer while-loops

1. Recognize that a range of integers b..c has to be processed
2. Write the command and equivalent postcondition
3. Write the basic part of the while-loop
4. Write loop invariant
5. Figure out any initialization
6. Implement the repetend (process k)

```
# Process b..c
Initialize variables (if necessary) to make invariant true
# Invariant: range b..k-1 has been processed
while k <= c:
    # Process k
    k = k + 1
# Postcondition: range b..c has been processed
```

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Finding an Invariant

Command to do something

```
# Make b True if n is prime, False otherwise
b = True
k = 2
# invariant: b is True if no int in 2..k-1 divides n, False otherwise
while k < n:
    # Process k;
    if n % k == 0:
        b = False
    k = k +1
# b is True if no int in 2..n-1 divides n, False otherwise
```

Equivalent postcondition

What is the invariant?

1 2 3 … k-1 k k+1 … n

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Finding an Invariant

```
# set x to # adjacent equal pairs in s
x = 0
# inv: x = # adjacent equal pairs in s[0..k-1]
while k < len(s):
    # Process k

    k = k + 1
# x = # adjacent equal pairs in s[0..len(s)-1]
```

Command to do something

for s = 'ebeee', x = 2

Equivalent postcondition

k: next integer to process.
What is initialization for k?

A: k = 0
B: k = 1
C: k = –1
D: I don't know

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Reason carefully about initialization

```
# s is a string; len(s) >= 1
# Set c to largest element in s
c = ??
k = ??
# inv: c is largest element in s[0..k–1]
while k < len(s):
    # Process k
    k = k+1
# c = largest char in s[0..len(s)–1]
```

Command to do something

Equivalent postcondition

1. What is the invariant?
2. How do we initialize c and k?

A: k = 0; c = s[0]
B: k = 1; c = s[0]
C: k = 1; c = s[1]
D: k = 0; c = s[1]
E: None of the above

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