

Lecture 14

Recursion

Announcements for Today

Assignments

- Assignment 3 now graded
 - **Mean** 96.3, **Median** 99
 - **Time:** 7.5 hr, **StdDev:** 3.5 hr
 - With 666 responses (nice!)
- Assignment 4 is now up!
 - Parts 1-3: Can do already
 - Part 4: Material from today
 - Part 5: Covered on Thursday
 - Due in two weeks

Other Announcements

- View the lesson videos
 - **Videos 17.1-16.5** for **today**
 - **Videos 17.6-17.11** next time
 - New videos posted Thursday
- Prelim to be graded **Saturday**
 - Will post grade in evening
 - Will give grade boundaries
 - In time for drop deadline
 - **But Bs are good grades!**

Activity Time: The Call Stack

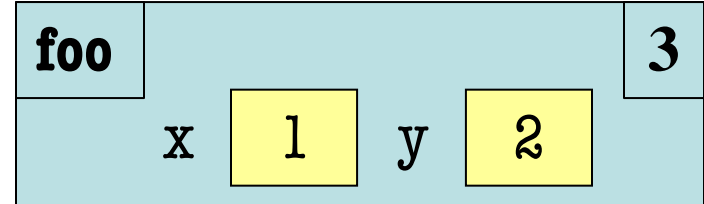
Function Definitions

```
1 def foo(x):  
2     y = x+1  
3     return bar(y+1)  
4  
5 def bar(y):  
6     return foo(y-1)
```

Function Call

```
>>> foo(1)
```

Assume we are here:



What is the **next step**?

Activity Time: The Call Stack

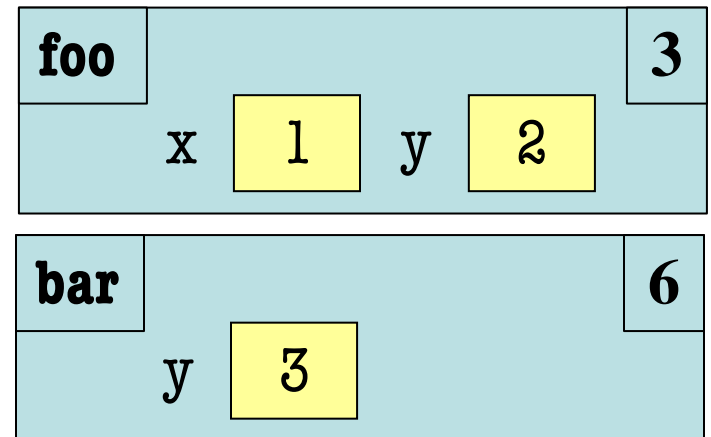
Function Definitions

```
1 def foo(x):  
2     y = x+1  
3     return bar(y+1)  
4  
5 def bar(y):  
6     return foo(y-1)
```

Function Call

>>> foo(1)

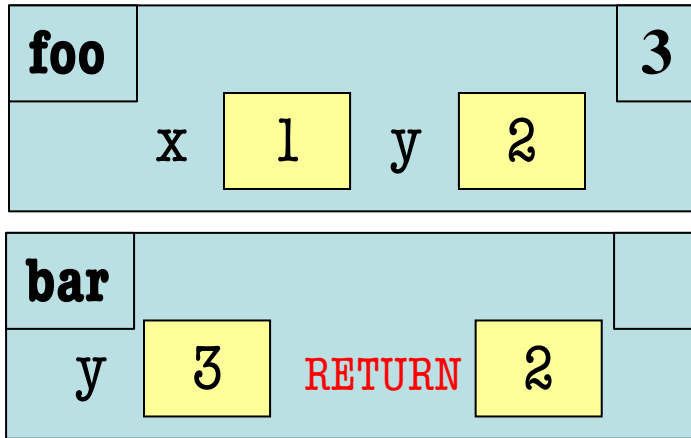
A:



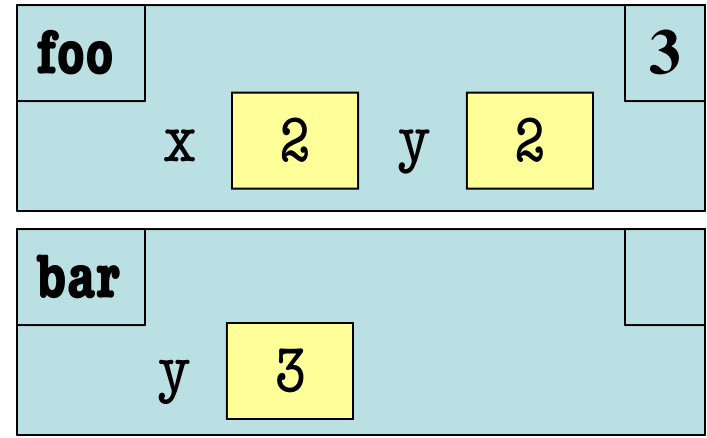
What is the **next step**?

Which One is Closest to Your Answer?

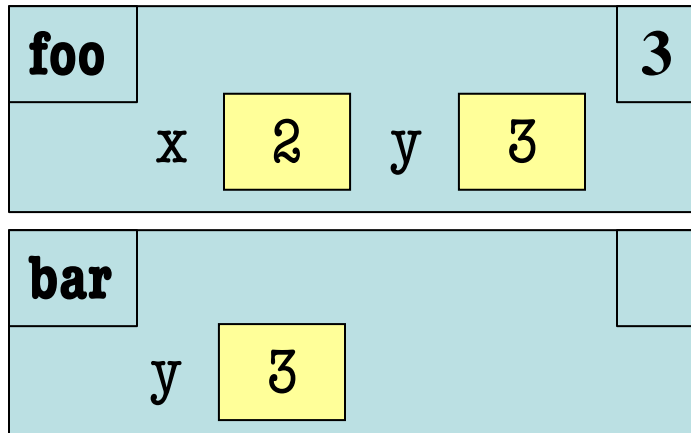
A:



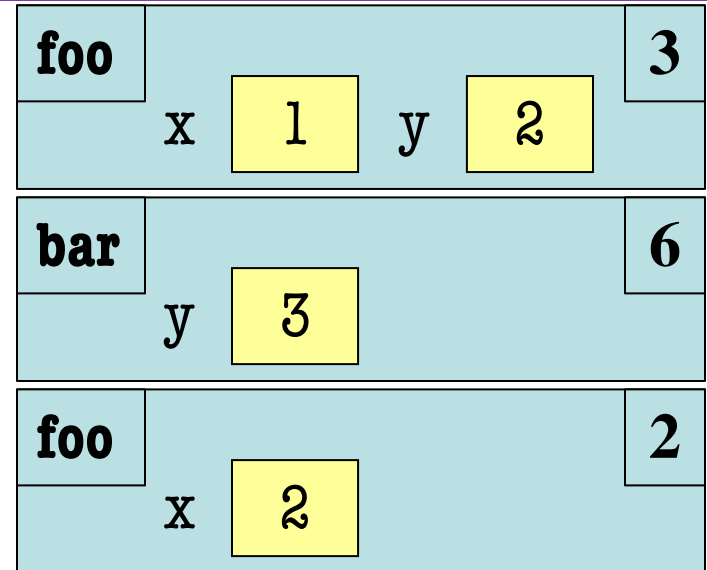
B:



C:



D:



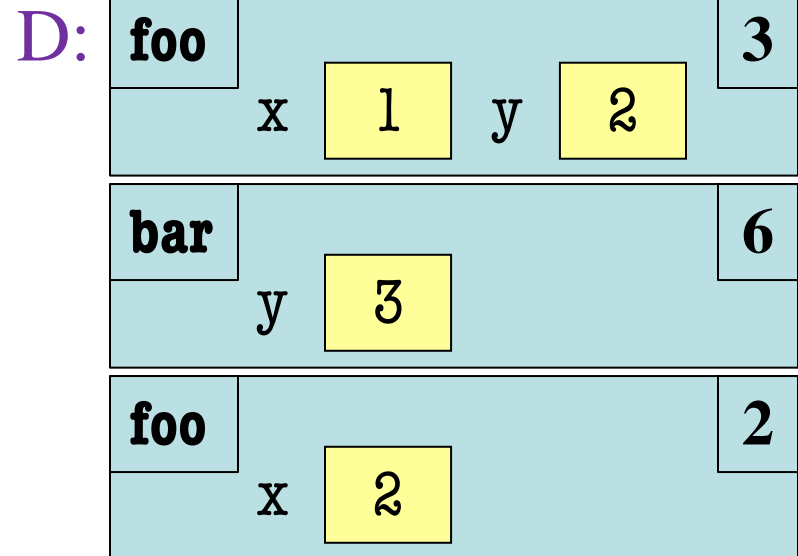
Activity Time: The Call Stack

Function Definitions

```
1 def foo(x):  
2     y = x+1  
3     return bar(y+1)  
4  
5 def bar(y):  
6     return foo(y-1)
```

Function Call

>>> foo(1)



This is called
Mutal Recursion

Simple Recursive Function

```
def lucas(n,p,q):
```

```
    """
```

```
    Returns the nth Lucas number for coefficients p and q.
```

```
    A Lucas number is a generalization of the Fibonacci Sequence.  
    The nth Lucas number L(n) is given by the recursive definition
```

$$L(0) = 0$$

$$L(1) = 1$$

$$L(n) = p * L(n-1) - q * L(n-2)$$

```
    Preconditions: n is an int >= 0, p and q are ints
```

```
    """
```

```
    pass
```

Simple Recursive Function

```
def lucas(n,p,q):
```

```
    """
```

```
    Returns the nth Lucas number for coef
```

```
    A Lucas number is a generalization of t
```

```
    The nth Lucas number L(n) is given by
```

$$L(0) = 0$$

$$L(1) = 1$$

$$L(n) = p * L(n-1) - q * L(n-2)$$

```
    Preconditions: n is an int >= 0, p is an
```

```
    """
```

```
    pass
```

Base Case?

A: $n = 0$

B: $n = 1$

C: $n = 0, n = 1$

D: $n = 0, p = 0$

E: $n = 0, p = 0, q = 0$

Simple Recursive Function

```
def lucas(n,p,q):
```

```
    """
```

```
    Returns the nth Lucas number for coef
```

```
    A Lucas number is a generalization of t
```

```
    The nth Lucas number L(n) is given by
```

$$L(0) = 0$$

$$L(1) = 1$$

$$L(n) = p * L(n-1) - q * L(n-2)$$

```
    Preconditions: n is an int >= 0, p is an
```

```
    """
```

```
    pass
```

Base Case?

A: $n = 0$

B: $n = 1$

C: $n = 0, n = 1$

D: $n = 0, p = 0$

E: $n = 0, p = 0, q = 0$

Divide and Conquer

```
def prod(tup):
```

```
    """
```

```
    Returns the product of the integers in tup. Returns 1 if empty.
```

```
    Examples:
```

```
        prod((12,)) returns 12
```

```
        prod((7,12,1,2,2)) returns 336
```

```
        prod(()) returns 1
```

```
    Precondition: tup is a tuple of ints
```

```
    """
```

```
    pass
```

Divide and Conquer

```
def prod(tup):
```

```
    """
```

```
    Returns the product of the integers in tup. Returns 1 if empty.
```

```
    Examples:
```

```
        prod((12,)) returns 12
```

```
        prod((7,12,1,2,2)) returns 336
```

```
        prod(()) returns 1
```

```
    Precondition: tup is a tuple of ints
```

```
    """
```

```
    pass
```

How Divide?

A: Cut in half

B: Pull off one elt.

C: Does not matter

Divide and Conquer

```
def prod(tup):
```

```
    """
```

```
    Returns the product of the integers in tup. Returns 1 if empty.
```

```
    Examples:
```

```
        prod((12,)) returns 12
```

```
        prod((7,12,1,2,2)) returns 336
```

```
        prod(()) returns 1
```

```
    Precondition: tup is a tuple of ints
```

```
    """
```

```
    pass
```

How Combine?

A: Add left, right

B: Multiply left, right

C: Does not matter

Divide and Conquer 2

```
def depunct(s):
```

```
    """
```

```
    Returns s but with everything that is not a letter removed
```

```
    Examples:
```

```
        depunct('Hello') returns 'Hello'
```

```
        depunct('Hello World!') returns 'HelloWorld'
```

```
    Parameter: s the string to edit
```

```
    Precondition s is a string
```

```
    """
```

```
    pass
```

Divide and Conquer 2

```
def depunct(s):
```

```
    """
```

```
    Returns s but with everything that is not a letter removed
```

```
    Examples:
```

```
        depunct('Hello') returns 'Hello'
```

```
        depunct('Hello World!') returns 'Hello World'
```

```
    Parameter: s the string to edit
```

```
    Precondition s is a string
```

```
    """
```

```
    pass
```

How Divide?

A: Cut in half

B: Pull off one elt.

C: Does not matter

Divide and Conquer 2

```
def depunct(s):
```

```
    """
```

```
    Returns s but with everything that is not a letter removed
```

```
    Examples:
```

```
        depunct('Hello') returns 'Hello'
```

```
        depunct('Hello World!') returns 'Hello World'
```

```
    Parameter: s the string to edit
```

```
    Precondition s is a string
```

```
    """
```

```
    pass
```

How Combine?

A: Add left, right

B: Add right, left

C: Does not matter

Divide and Conquer 3

```
def reverse(s):
```

```
    """
```

```
    Returns s with its characters in reverse order
```

```
    Examples:
```

```
        depunct('Hello') returns 'olleH'
```

```
        depunct('amma') returns 'amma'
```

```
    Parameter: s the string to reverse
```

```
    Precondition s is a string
```

```
    """
```

```
    pass
```

Divide and Conquer 3

```
def reverse(s):
```

```
    """
```

```
    Returns s with its characters in reverse order
```

```
    Examples:
```

```
        depunct('Hello') returns 'olleH'
```

```
        depunct('amma') returns 'amma'
```

```
    Parameter: s the string to reverse
```

```
    Precondition s is a string
```

```
    """
```

```
    pass
```

How Divide?

A: Cut in half

B: Pull off one elt.

C: Does not matter

Divide and Conquer 3

```
def reverse(s):
```

```
    """
```

```
    Returns s with its characters in reverse order
```

```
    Examples:
```

```
        depunct('Hello') returns 'olleH'
```

```
        depunct('amma') returns 'amma'
```

```
    Parameter: s the string to reverse
```

```
    Precondition s is a string
```

```
    """
```

```
    pass
```

How Combine?

A: Add left, right

B: Add right, left

C: Does not matter

Questions?