http://www.cs.cornell.edu/courses/cs1110/2021sp

Lecture 16: More Recursion!

CS 1110

Introduction to Computing Using Python

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S. Marschner, C. Van Loan, W. White]

Announcements

- Prelim 1 accounts for 15% of course grade only. Treat it as a diagnostic tool: is there a topic that you need to review? Strengthen your foundation now. 1-on-1 meeting opportunities to be available on CMS soon
- Attend your lab session! *New experiment:* you can additionally attend another online lab session to get more help on weekly lab exercises
- Assignment 4 to be released after lecture. Due Apr 13.
- ACSU annual Research Night, Apr 8 5:30-7:30pm
  - Interested in undergraduate research in CS?
  - https://discord.com/invite/cCM3QuGY3B

Recursion

Recursive Function:

A function that calls itself (directly or indirectly)

Recursive Definition:

A definition that is defined in terms of itself

From previous lecture: Factorial

Non-recursive definition:

$n! = n \times n\text{-}1 \times \ldots \times 2 \times 1$

$= n\,(n\text{-}1 \times \ldots \times 2 \times 1)$

Recursive definition:

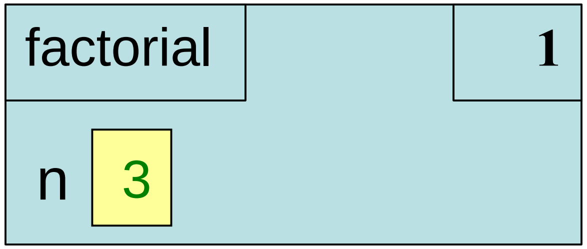
$n! = n\,(n\text{-}1)!$ for $n > 0$ **Recursive case**

$0! = 1$ **Base case**

Recursive Call Frames

```
def factorial(n):
    """Returns: factorial of n.
    Precondition: n ≥ 0 an int"""
1   if n == 0:
2       return 1

3   return n*factorial(n-1)
```



factorial(3)

Recursive Call Frames

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  def factorial(n):
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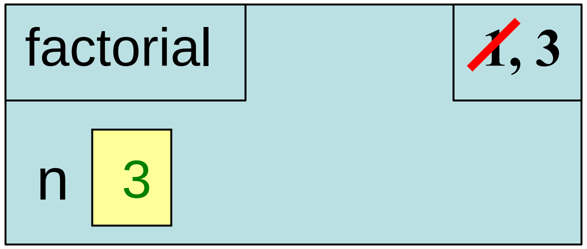

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Recursion

```
  def factorial(n):
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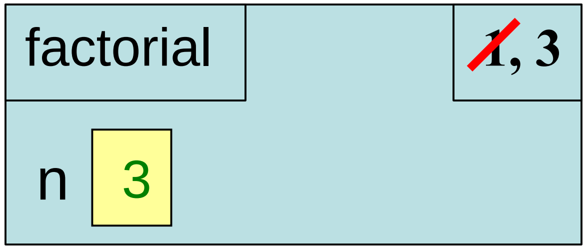
factorial(3)



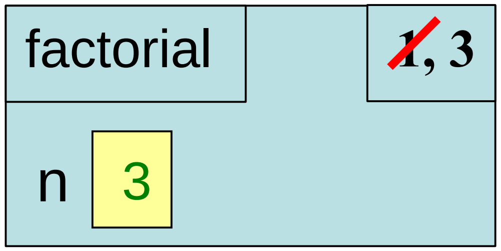
Now what?
Each call is
a new frame

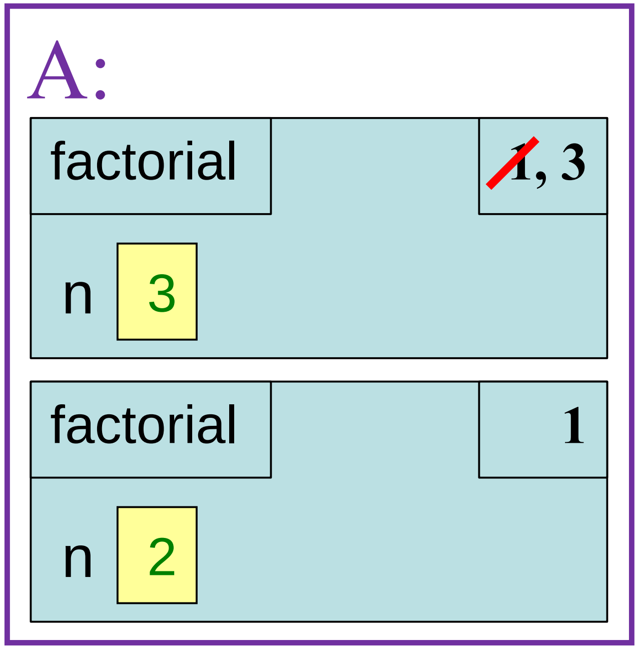
What happens next? (Q)

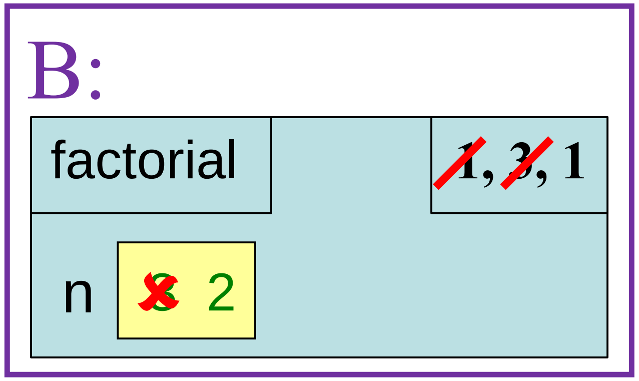
```
def factorial(n):
    """Returns: factorial of n.
    Pre: n ≥ 0 an int"""
    if n == 0:
        return 1

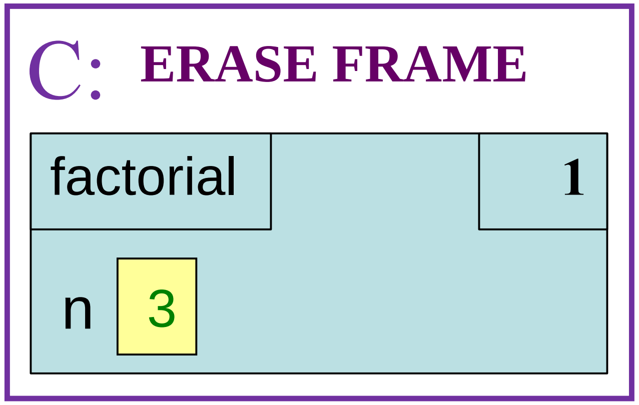
    return n*factorial(n-1)
```

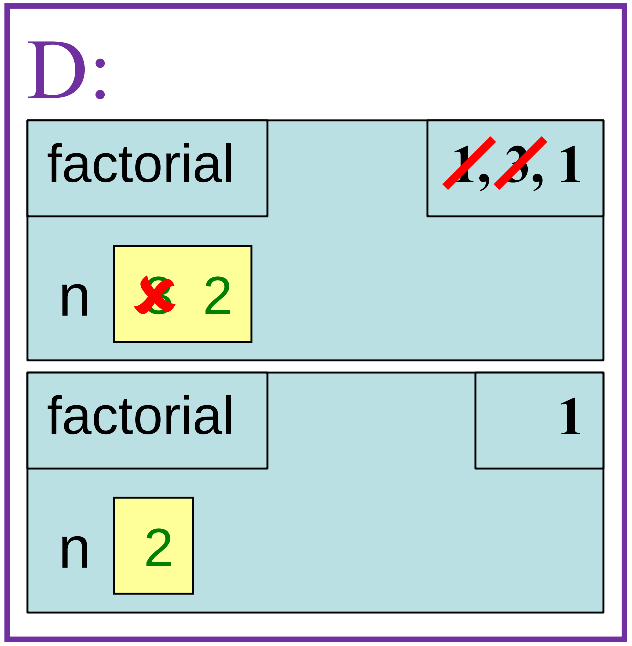
Call: factorial(3)











What happens next? (A)

```
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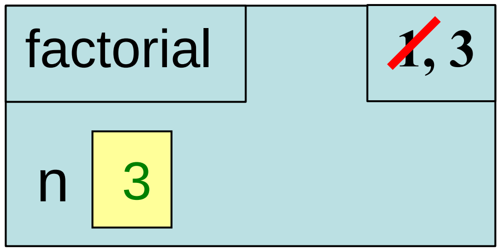
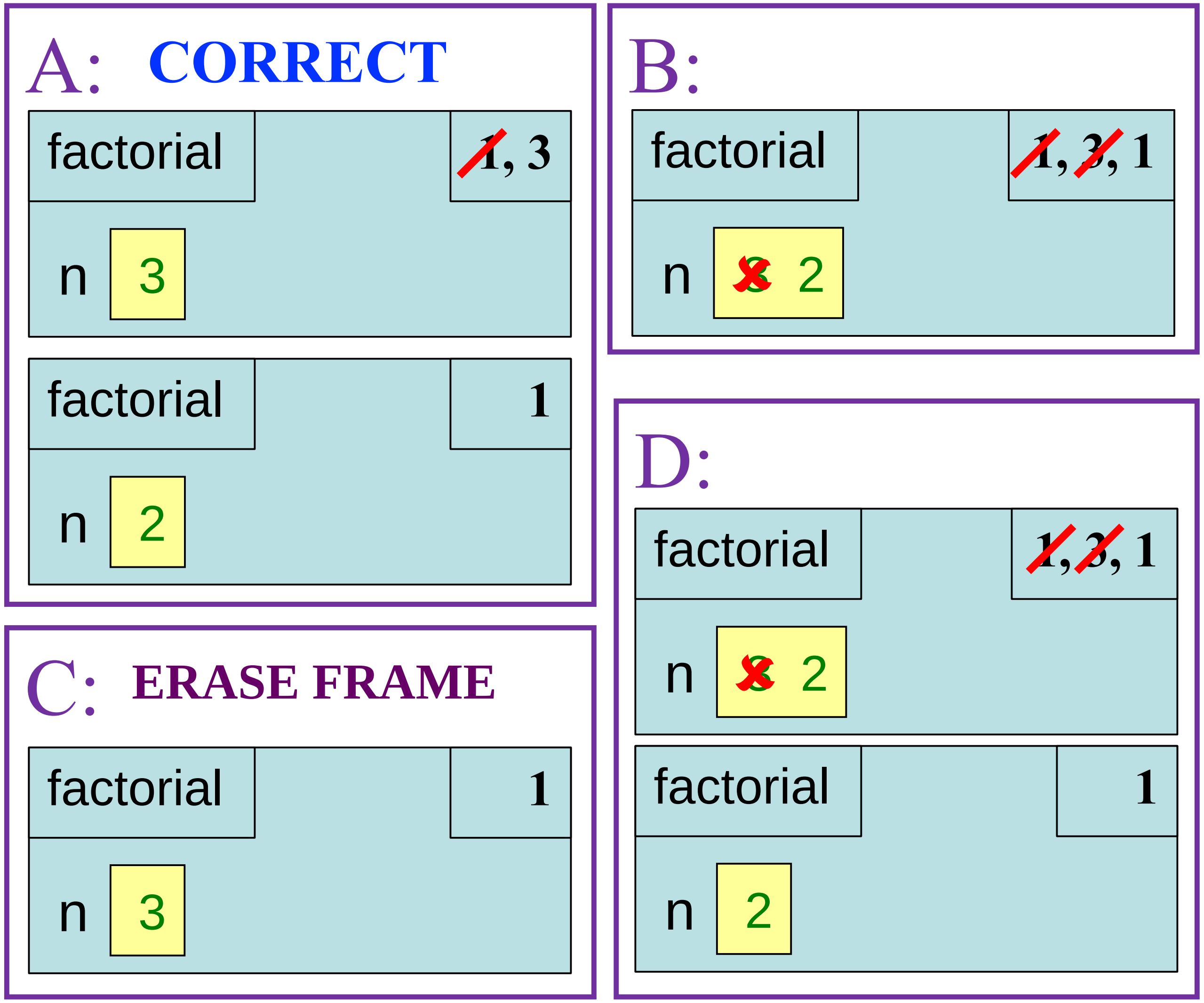
```

Call: factorial(3)





Recursive Call Frames

```
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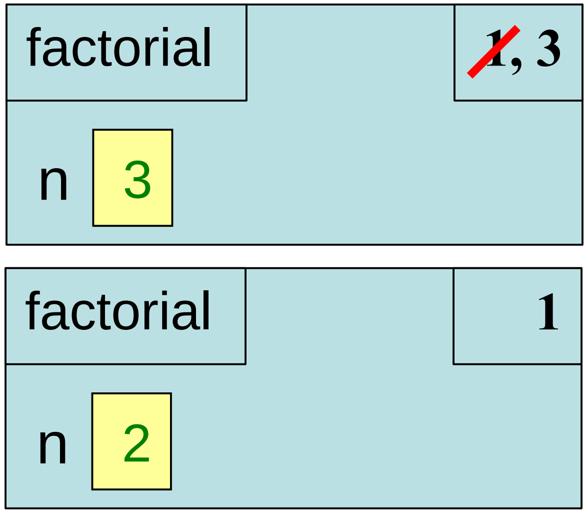
factorial(3)



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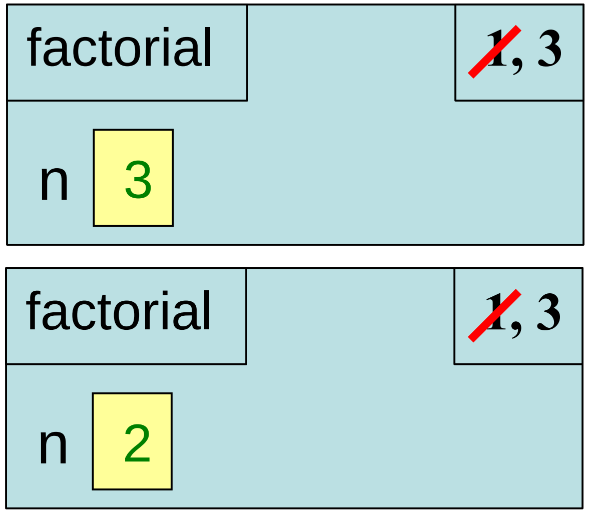

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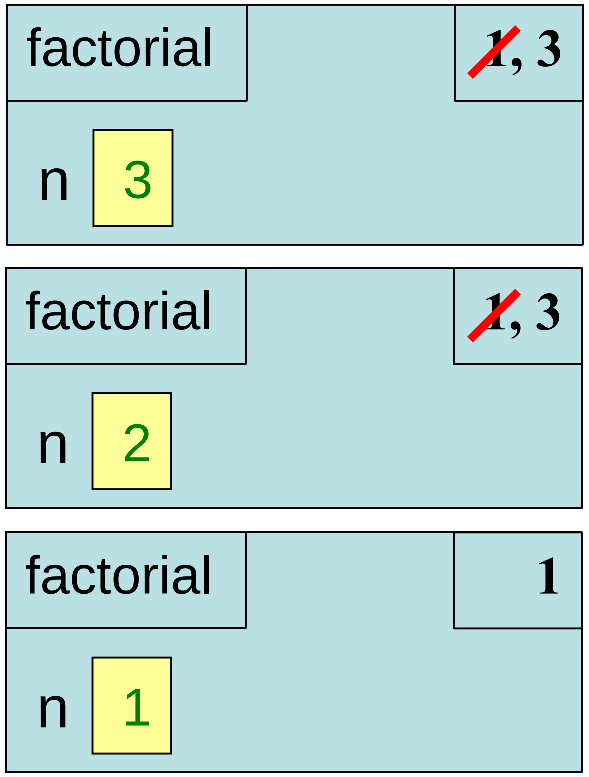

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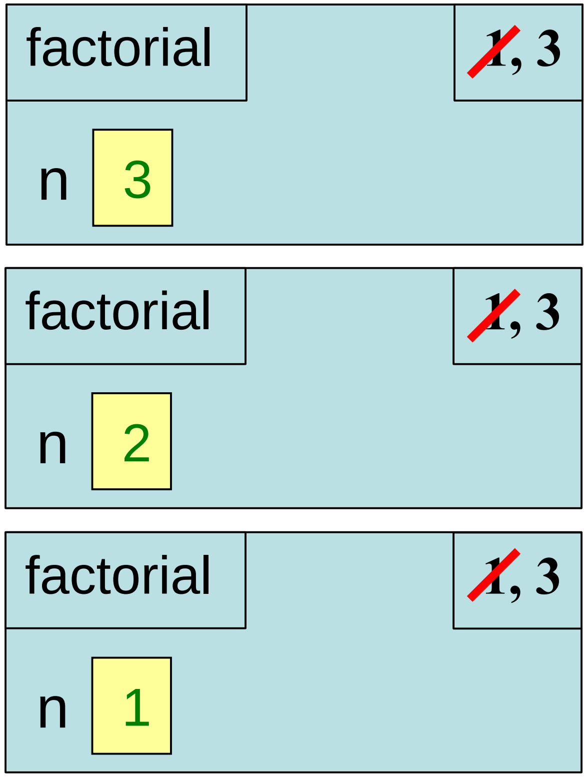

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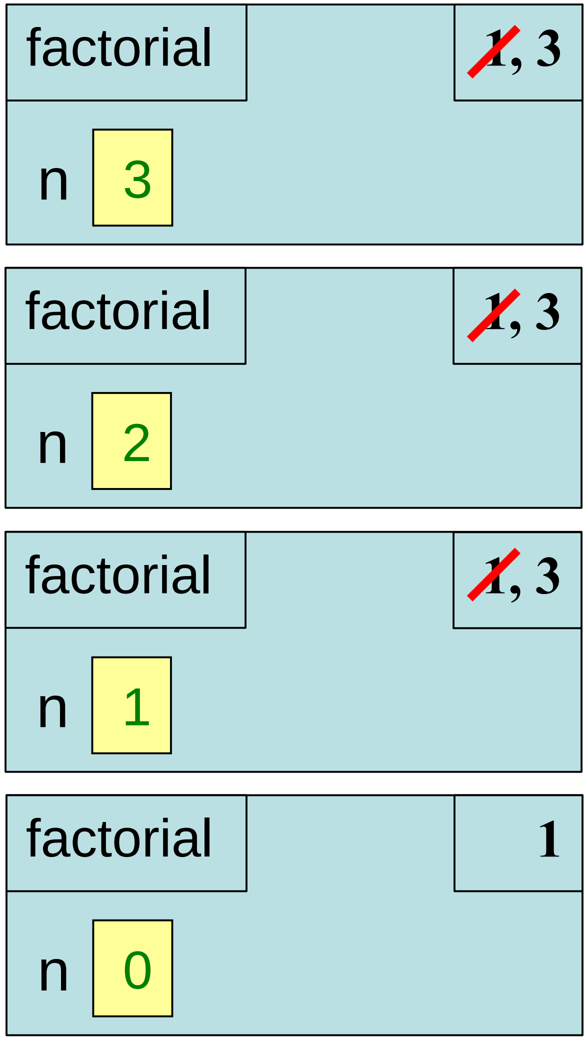

Recursive Call Frames

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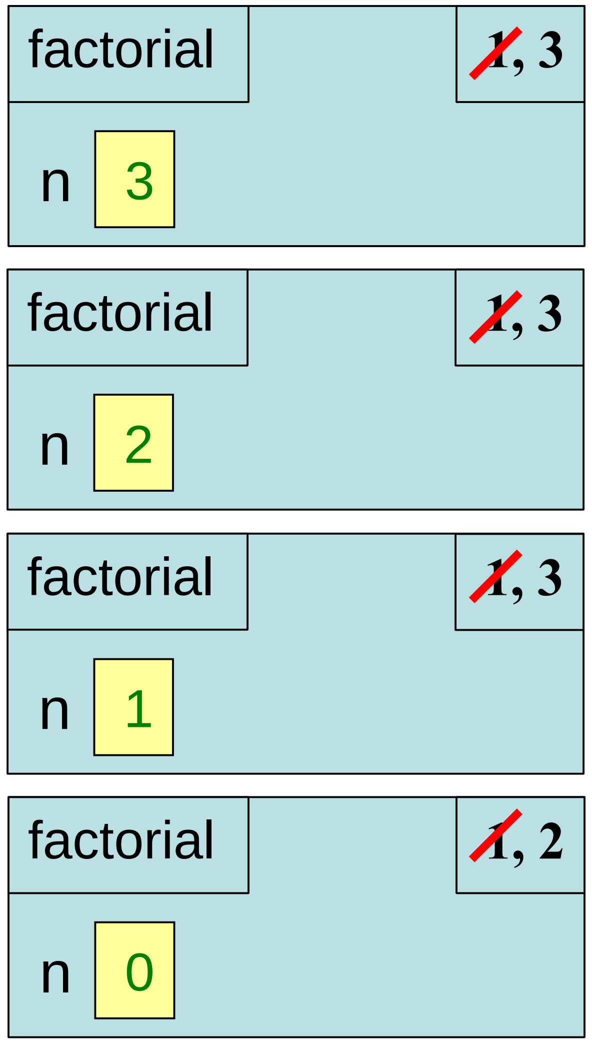

Recursive Call Frames

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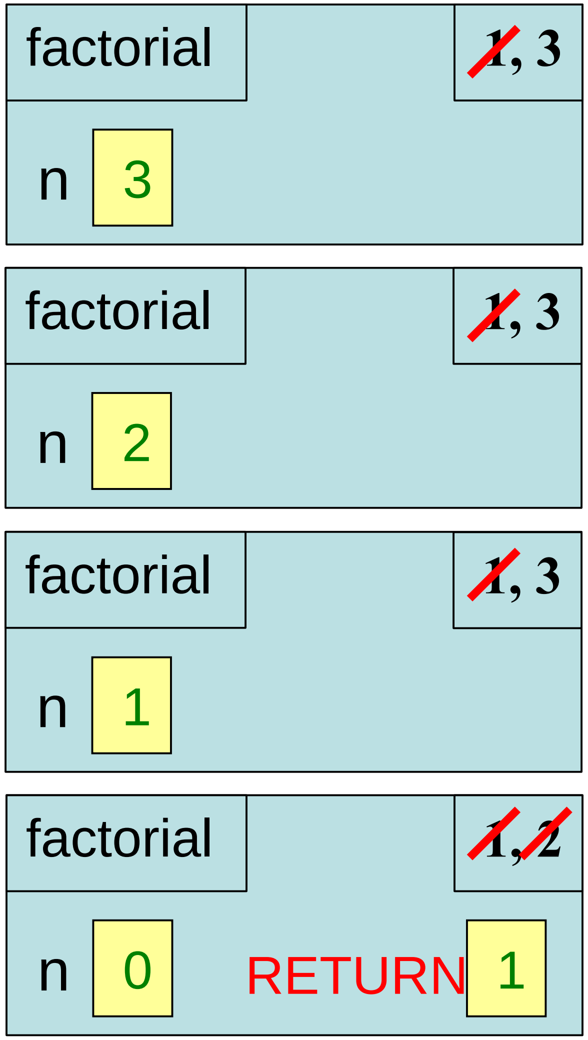

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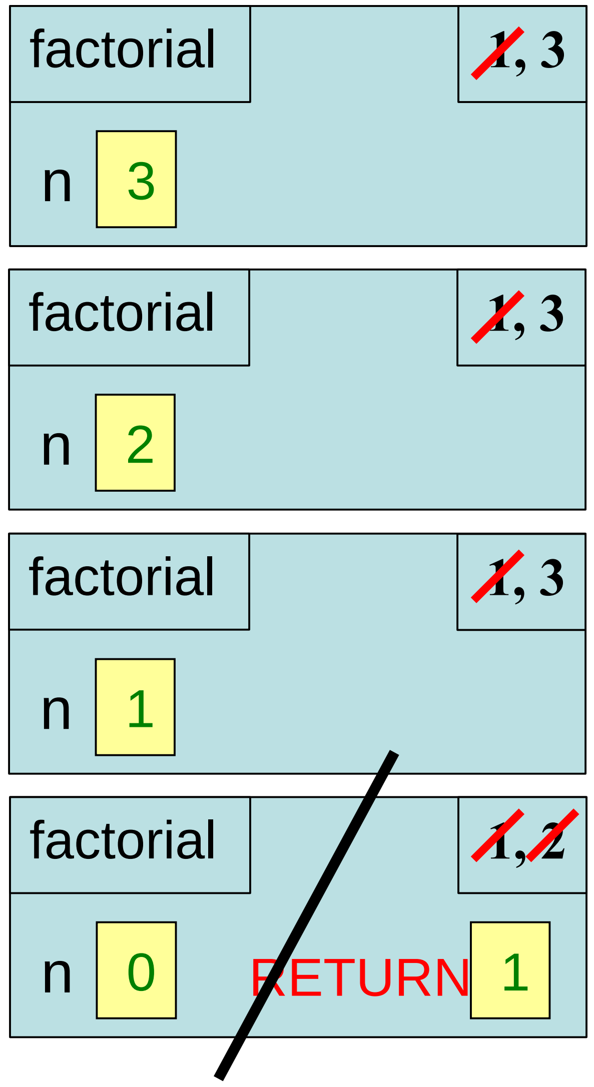

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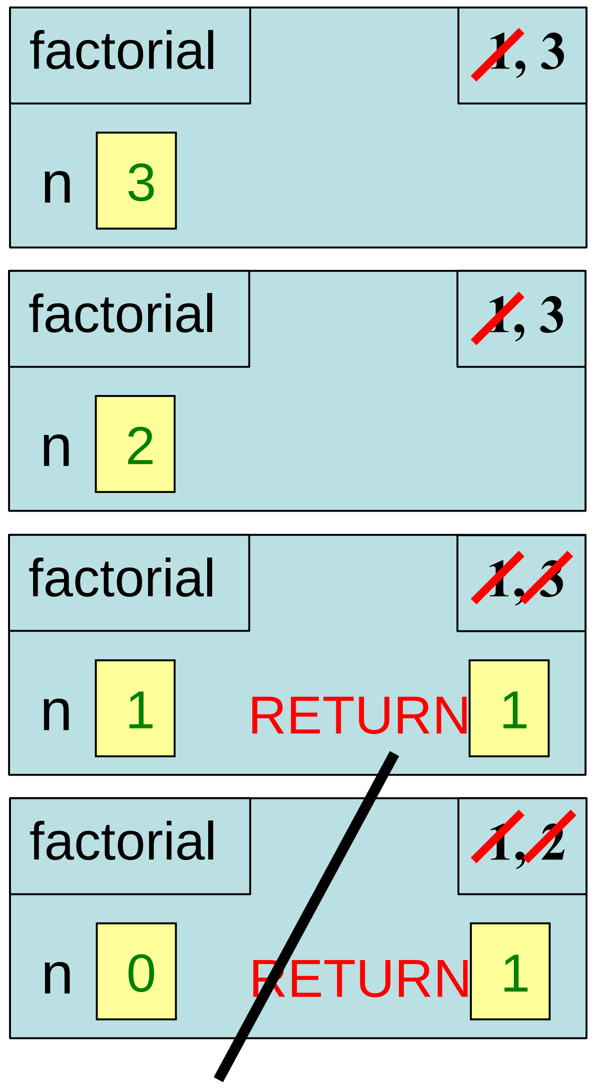

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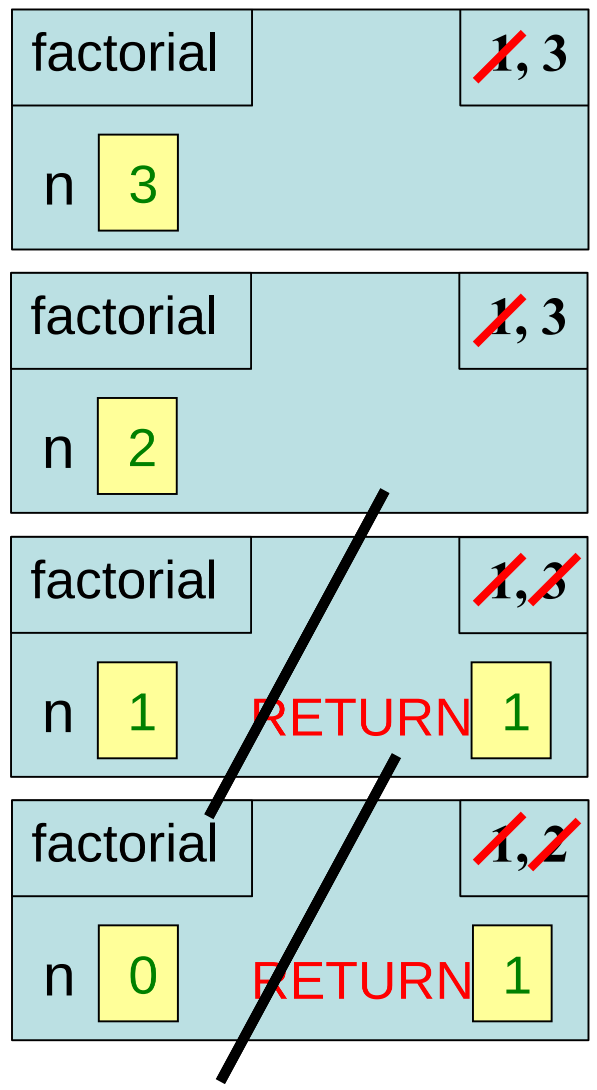

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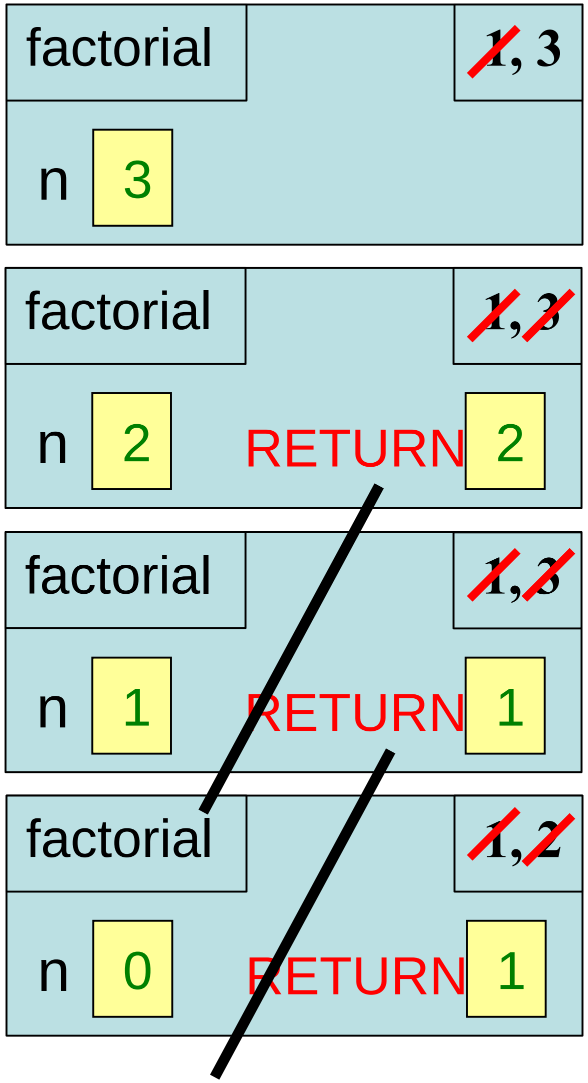

Recursive Call Frames

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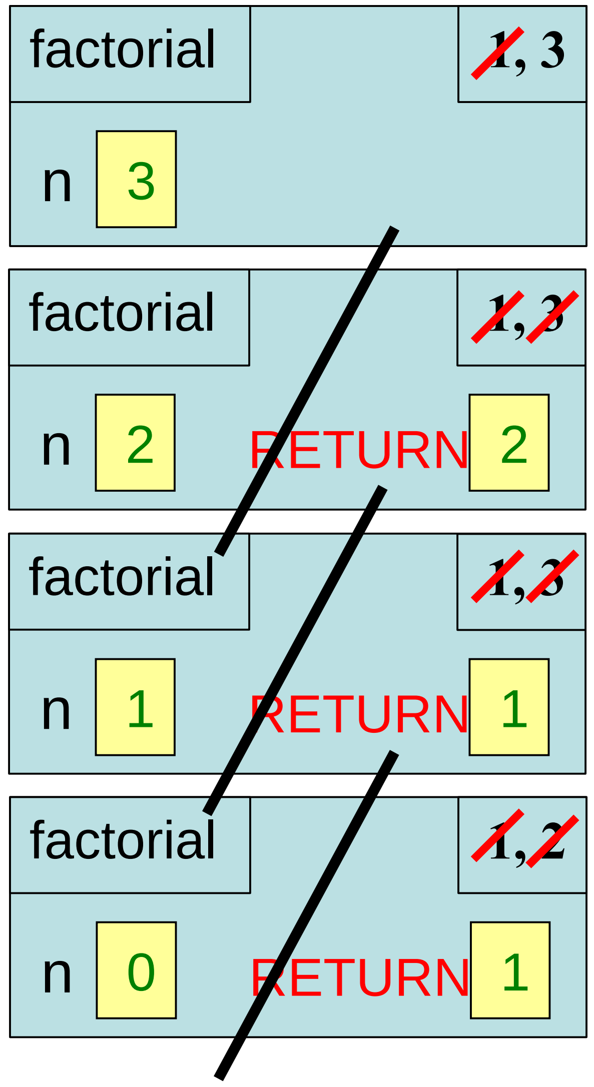

Recursive Call Frames

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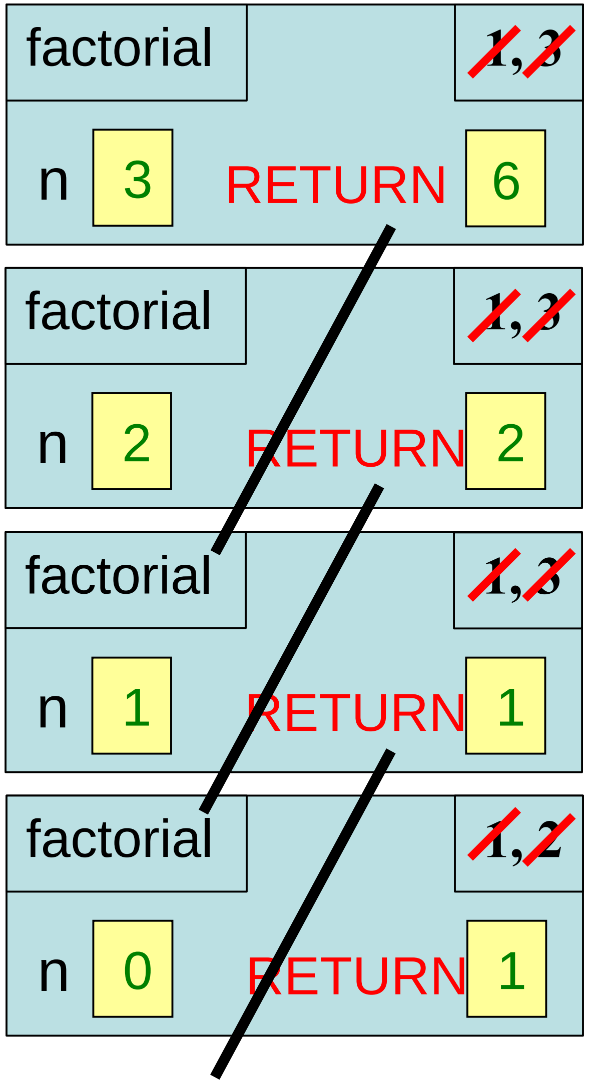

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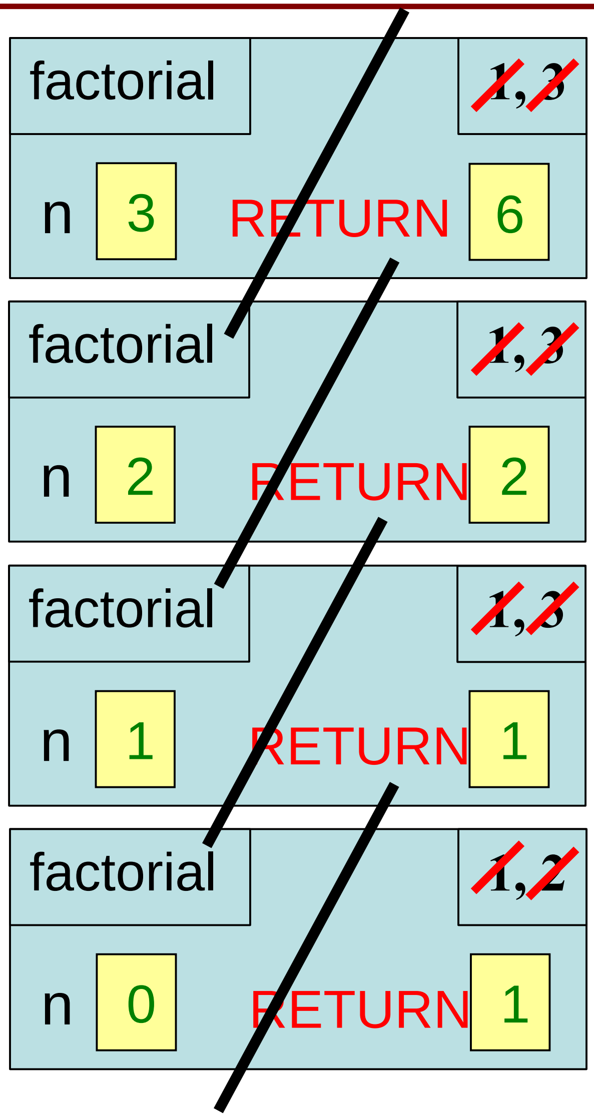
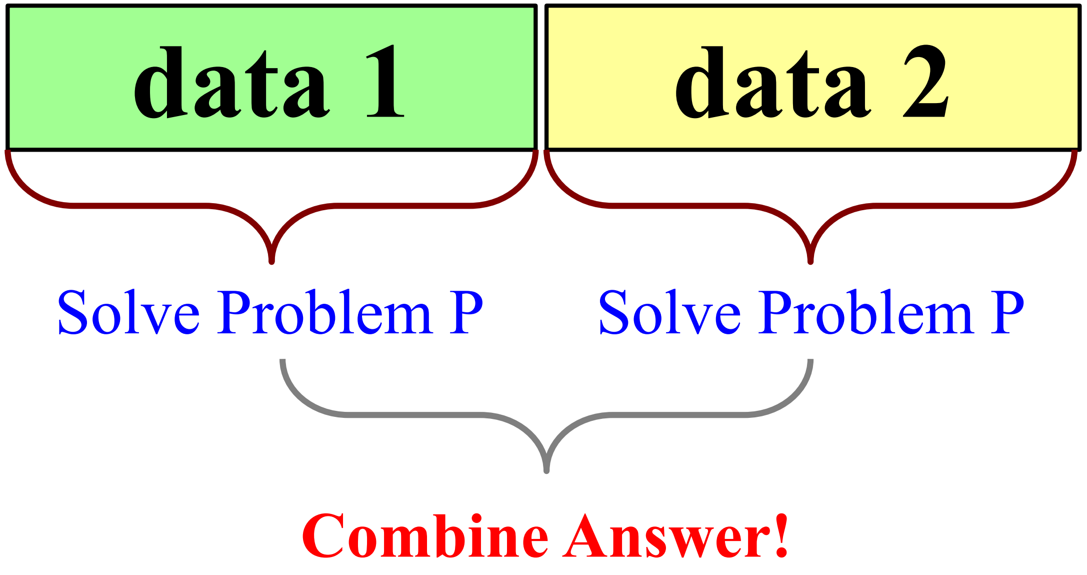


Divide and Conquer

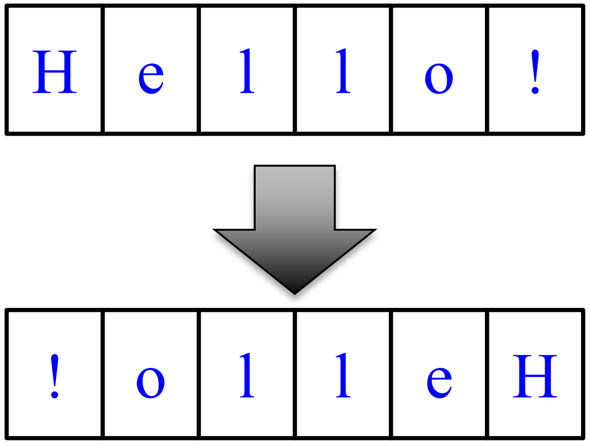
Goal: Solve problem P on a piece of data

Idea: Split data into two parts and solve problem



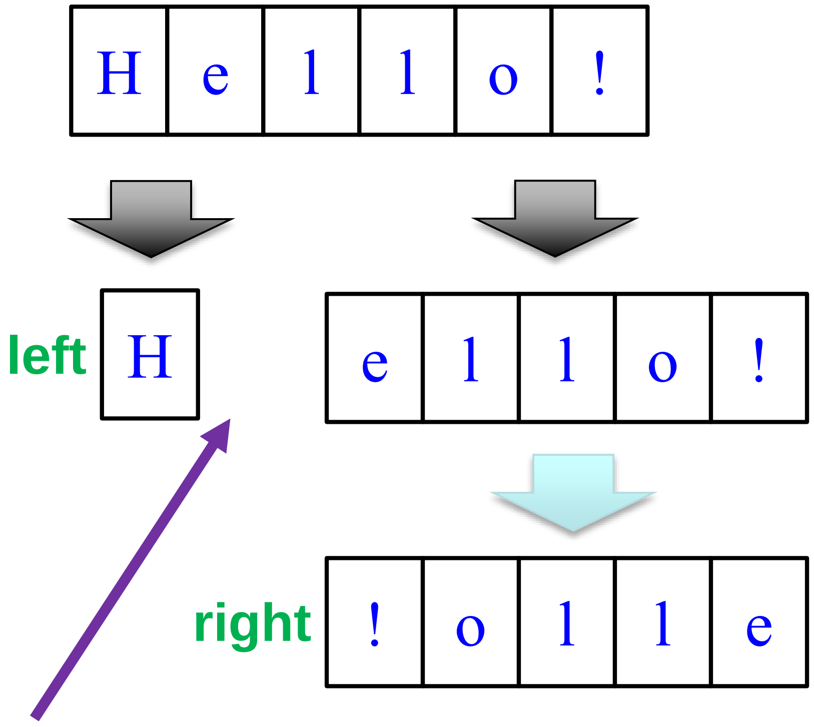
Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case



    # 2. Break into two parts



    # 3. Combine the result
```



Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case



    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
```

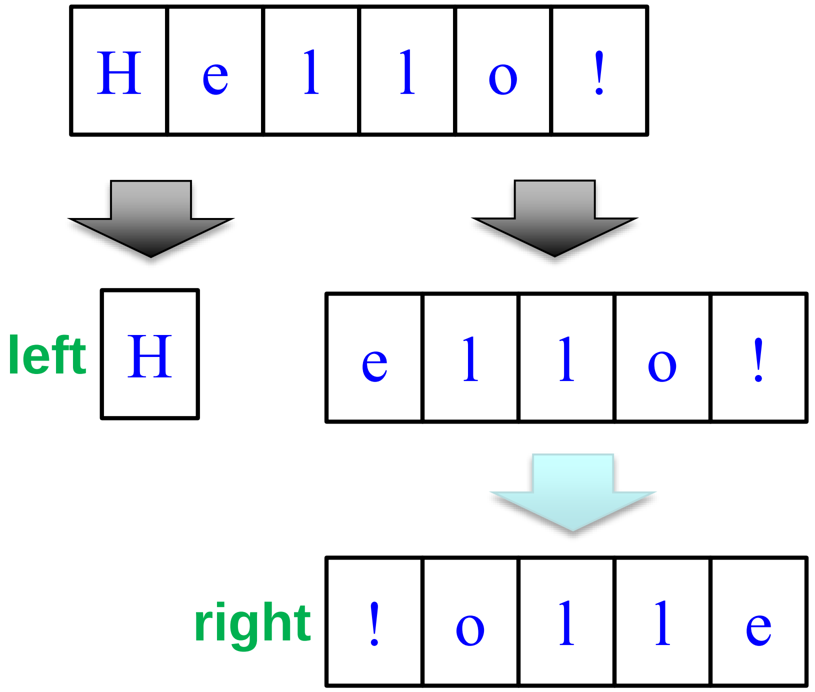


If this is how we break it up….

How do we combine it?

How to Combine? (Q)

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case



    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
    return
```



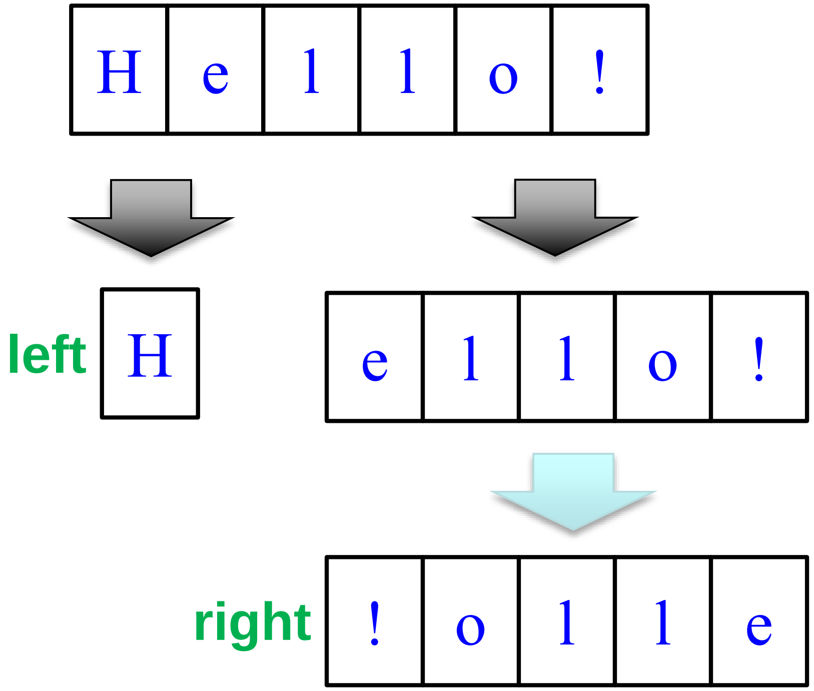
A: left + right | B: right + left | C: left | D: right

How to Combine? (A)

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case



    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
    return
```

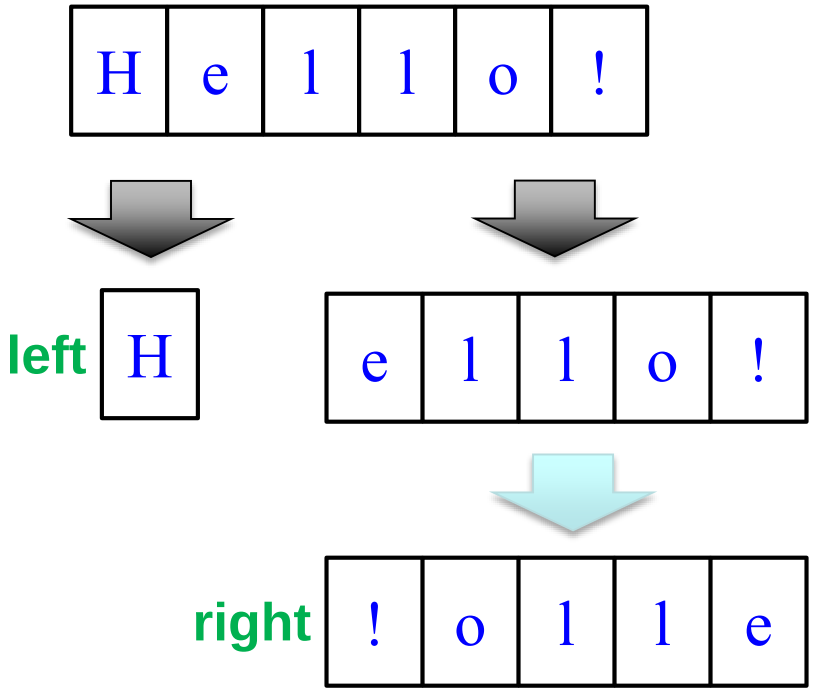
CORRECT

A: left + right	B: right + left	C: left	D: right



Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case



    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
     return right+left
```



What is the Base Case? (Q)

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case
```

H	e	l	l	o	!

```
A: if s == "":
       return s
```

```
B: if len(s) <= 2:
       return s
```

```
C: if len(s) <= 1:
       return s
```

```
    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
    return right+left
```

D: Either A or C would work

E: A, B, and C would all work

What is the Base Case? (A)

H	e	l	l	o	!

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case
```

CORRECT

```
A: if s == "":
       return s
```

```
B: if len(s) <= 2:
       return s
```

```
C: if len(s) <= 1:
       return s
```

```
    # 2. Break into two parts
    left   = reverse(s[0])
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    # 3. Combine the result
    return right+left
```

D: Either A or C would work

E: A, B, and C would all work

Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case
    if len(s) <= 1:
        return s

    # 2. Break into two parts
    left   = ~~reverse(s[0])~~    s[0]
    right = reverse(s[1:])

    # 3. Combine the result
    return right+left
```

Base Case

Recursive Case

Alternate Implementation (Q)

```
def reverse(s):
    """Returns: reverse of s
    Precondition: s a string"""
    # 1. Handle base case
    if len(s) <= 1:
        return s

    # 2. Break into two parts
    half   = len(s)//2
    left   = reverse(s[:half])
    right = reverse(s[half:])

    # 3. Combine the result
    return right+left
```

Does this work?

A: **YES**

B: **NO**

Alternate Implementation (A)

```
def reverse(s):
    """Returns: reverse of s
    Precondition: s a string"""
    # 1. Handle base case
    if len(s) <= 1:
        return s

    # 2. Break into two parts
    half   = len(s)//2
    left   = reverse(s[:half])
    right = reverse(s[half:])

    # 3. Combine the result
    return right+left
```

Does this work?

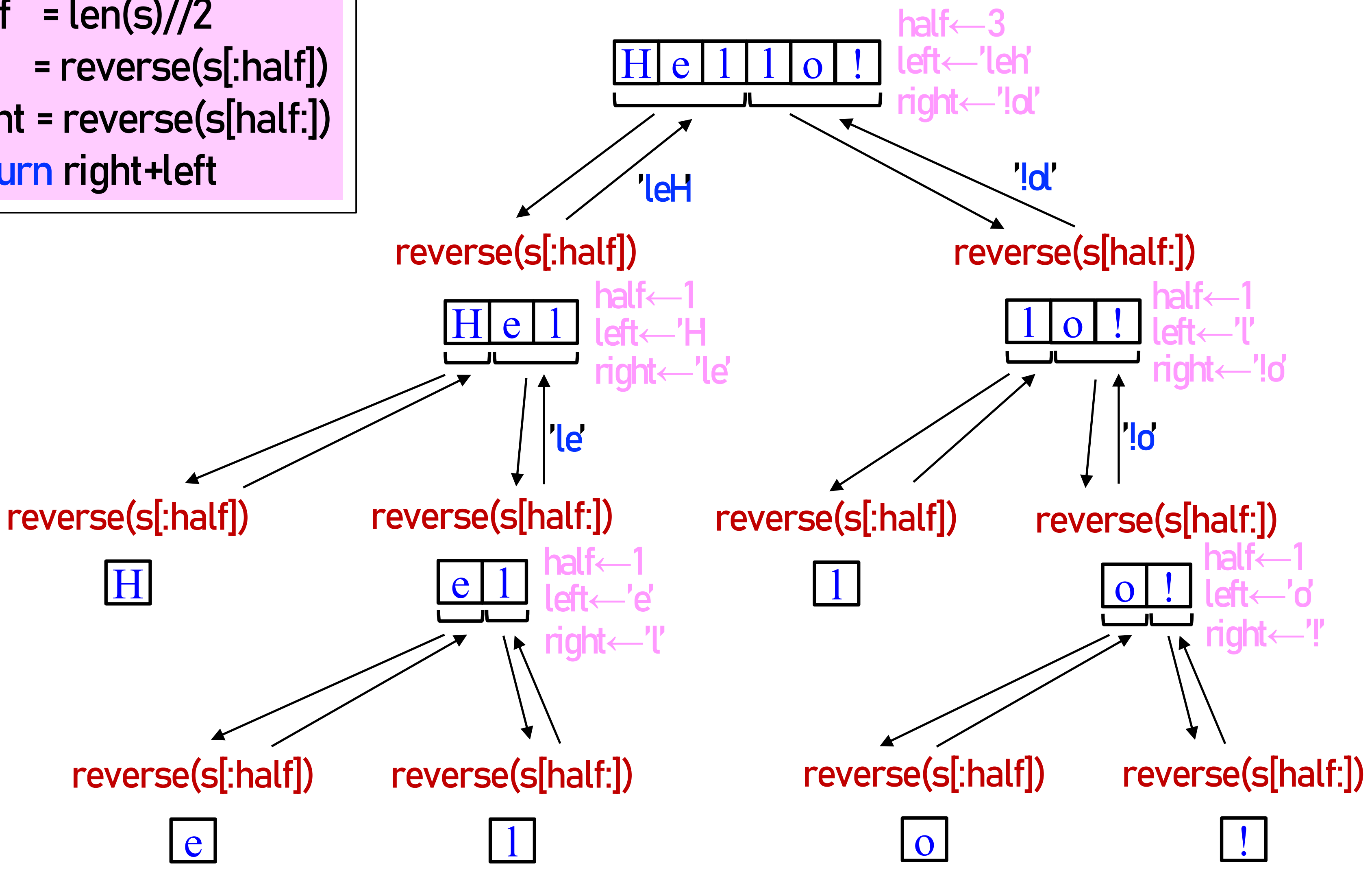
CORRECT [✓ A: YES]

B: NO

```
def reverse(s):
    if len(s) <= 1:
        return s
    half  = len(s)//2
    left  = reverse(s[:half])
    right = reverse(s[half:])
    return right+left
```

Execute the function call reverse('Hello!')

Result: '!olleh'



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def reverse(s):
    if len(s) <= 1:
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    half  = len(s)//2
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Execute the function call reverse('Hello!')

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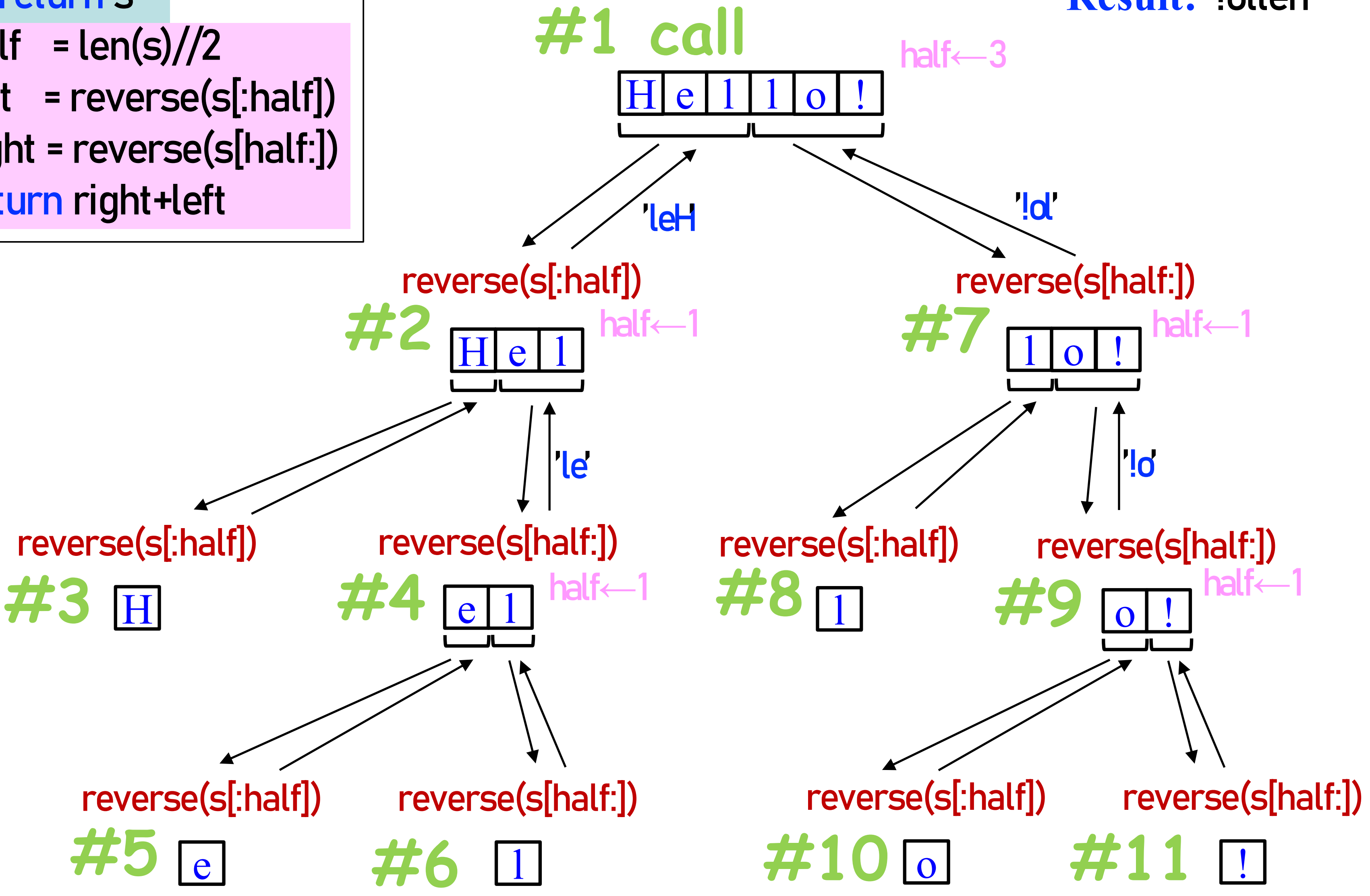
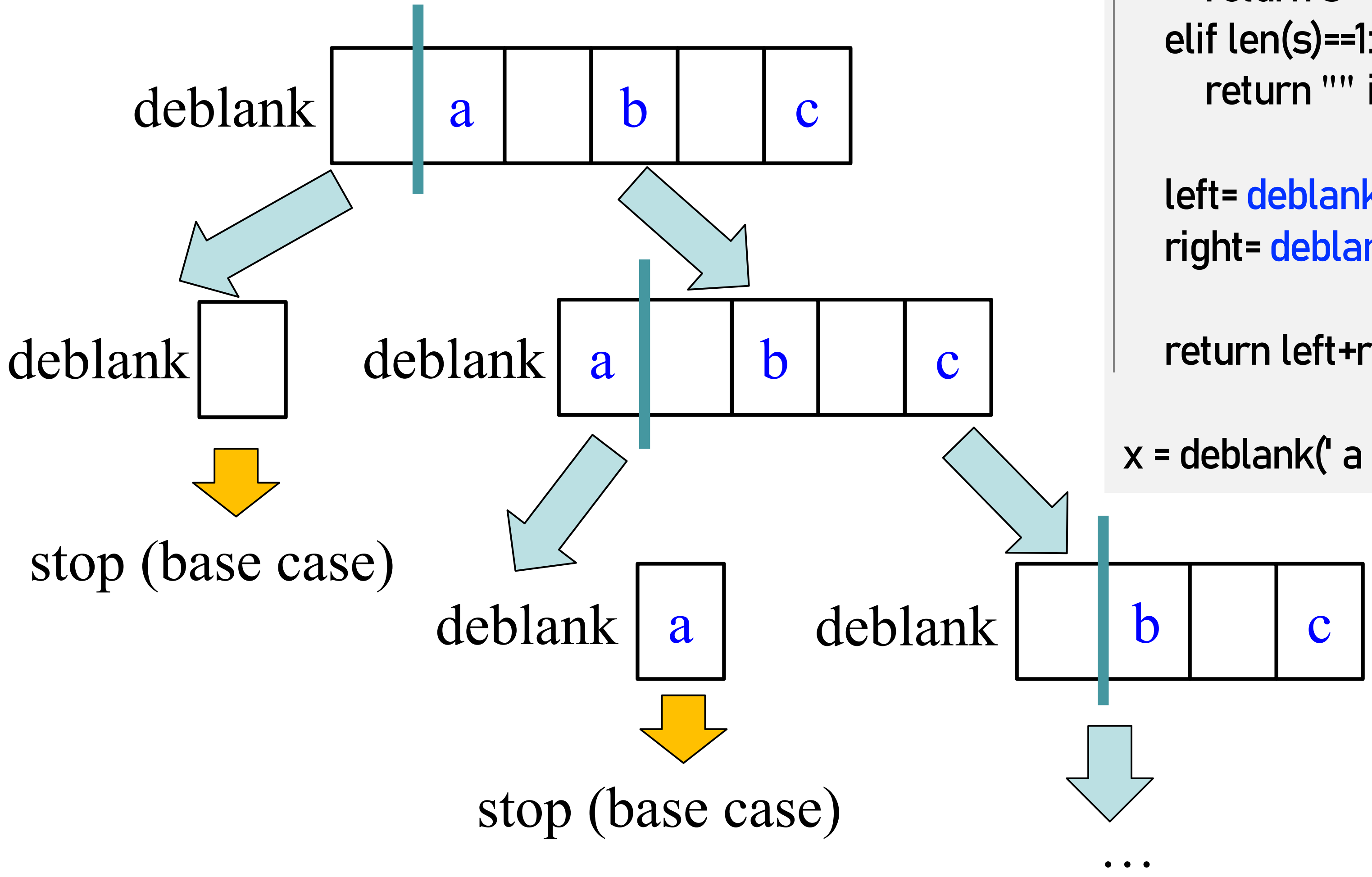


Following the Recursion



```
def deblank(s):
    """ Returns: s without spaces """
    if s == "":
        return s
    elif len(s)==1:
        return "" if s[0]==" " else s

    left= deblank(s[0])
    right= deblank(s[1:])

    return left+right

x = deblank(' a b c')
```

From last lecture: did you **visualize a call of `deblank` using Python Tutor**? Pay attention to the recursive calls (call frames opening up), the completion of a call (sending the result to the call frame "above"), and the resulting accumulation of the answer.

Example: Palindromes

- **Example:**

AMANAPLANACANALPANAMA

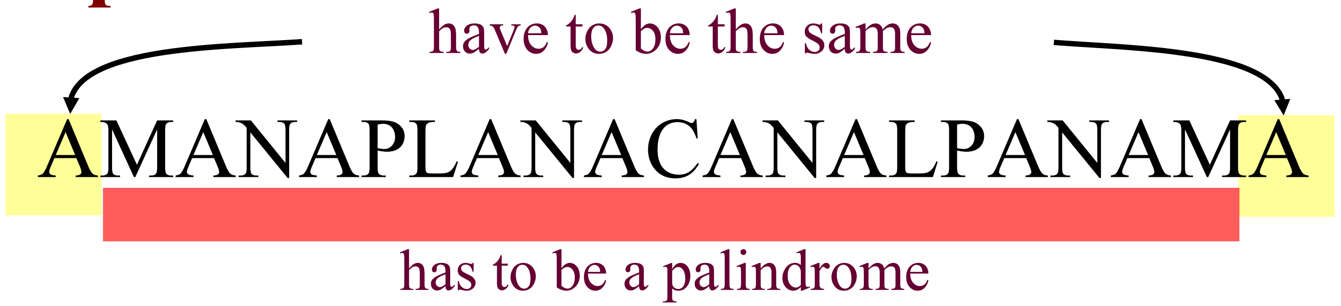
MOM

- Dictionary definition: “a word that reads (spells) the same backward as forward”

- Can we define recursively?

Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
  - its first and last characters are equal, and
  - the rest of the characters form a palindrome
- **Example:**



- **Implement:** def ispalindrome(s):

```
    """Returns: True if s is a palindrome"""
```

Example: Palindromes

String with ≥ 2 characters is a palindrome if:

- its first and last characters are equal, and
- the rest of the characters form a palindrome

Recursive Definition

```
def ispalindrome(s):
    """Returns: True if s is a palindrome"""
    if len(s) < 2:
        return True

    endsAreSame = ______________________________
    middleIsPali = _____________________________
    return ________________________________
```

Base case

Example: Palindromes

String with ≥ 2 characters is a palindrome if:

- its first and last characters are equal, and
- the rest of the characters form a palindrome

Recursive Definition

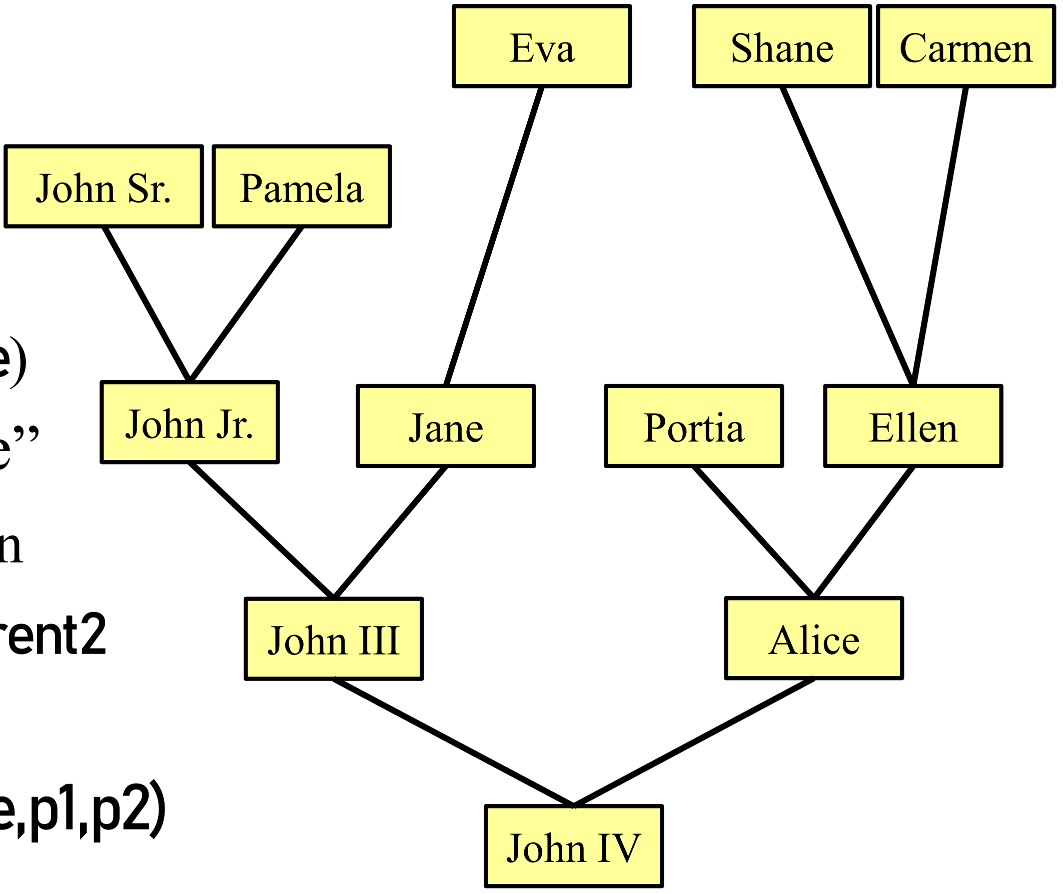
```
def ispalindrome(s):
    """Returns: True if s is a palindrome"""
    if len(s) < 2:
        return True

    endsAreSame = s[0] == s[-1]
    middleIsPali = ispalindrome(s[1:-1])
    return endsAreSame and middleIsPali
```

Base case

Recursive case

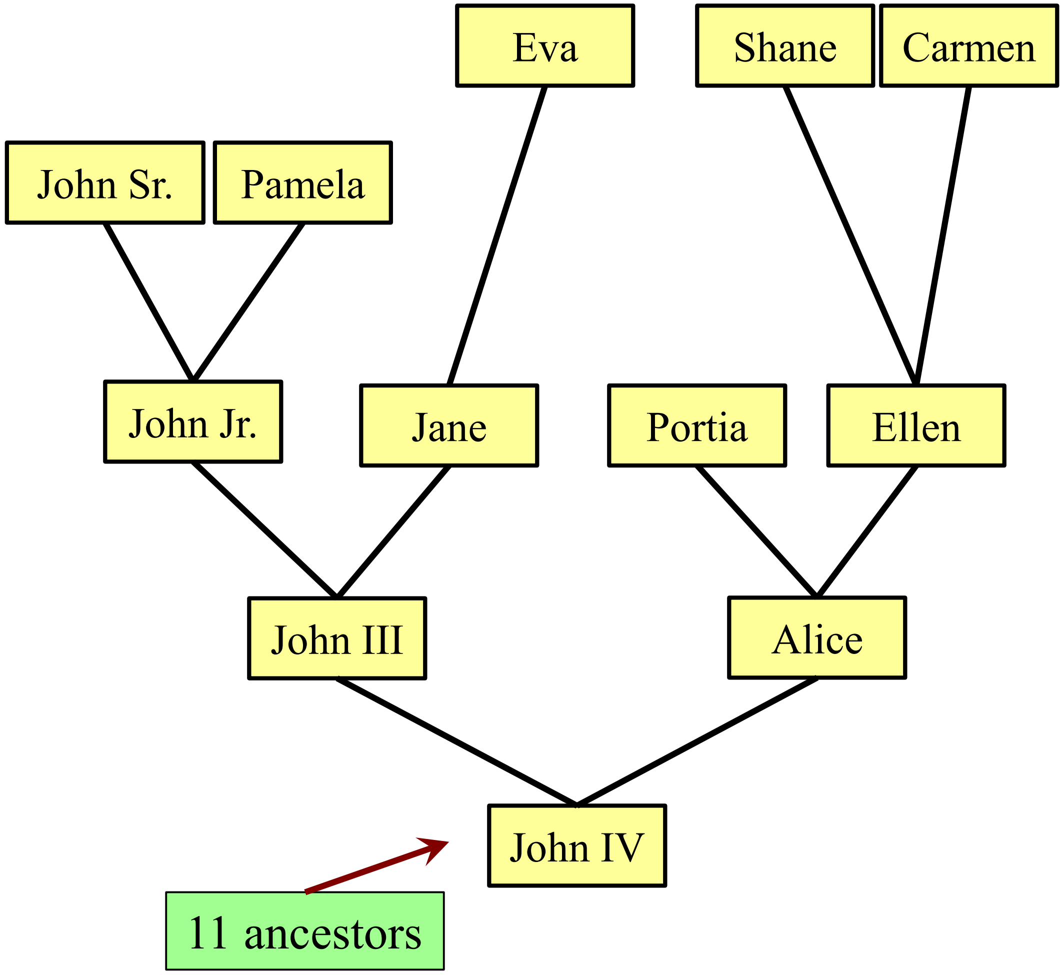
Recursion and Objects

- Class Person
  - Objects have 3 attributes
  - name: String
  - parent1: Person (or None)
  - parent2: Person (or None)
- Represents the "family tree"
  - Goes as far back as known
  - Attributes parent1 and parent2 are None if not known
- **Constructor**: Person(name,p1,p2)



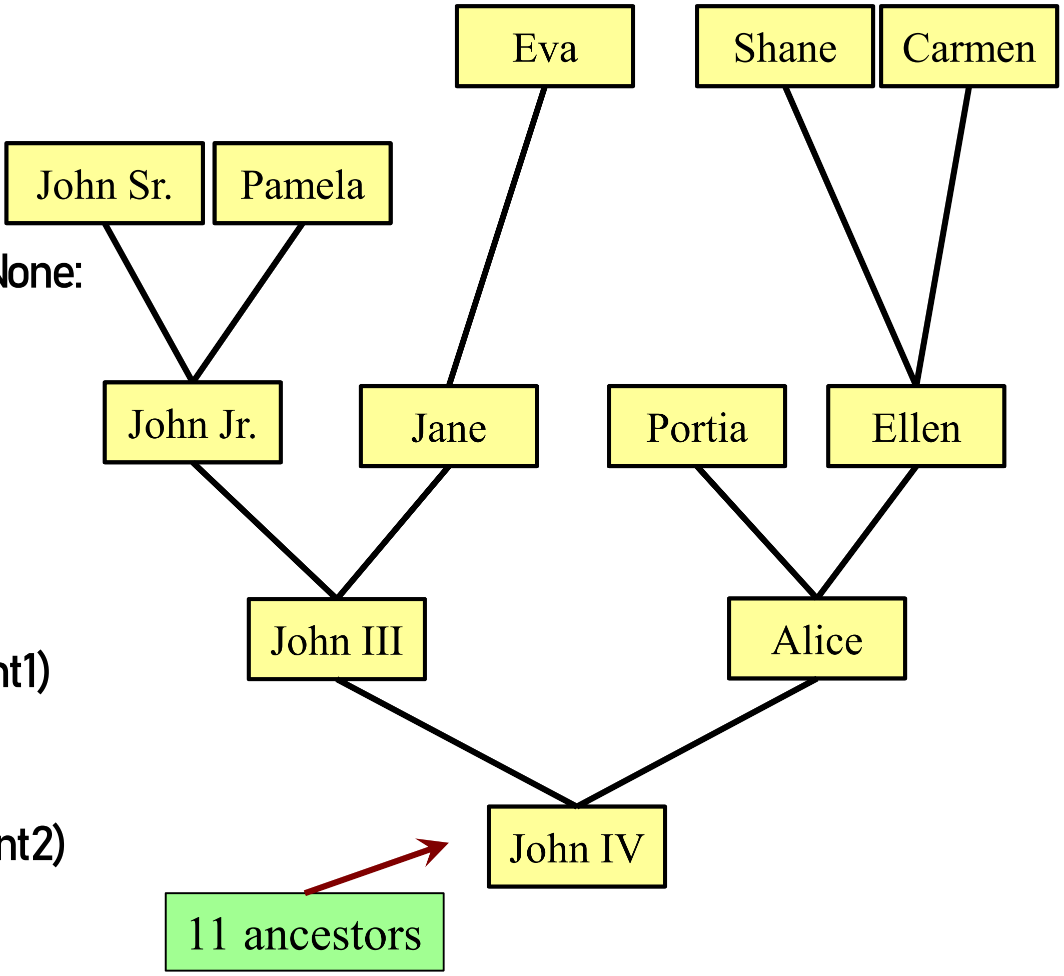
Recursion and Objects

```
def num_ancestors(p):
    """Returns: num of known ancestors
    Pre: p is a Person"""
    # 1. Handle base case.
    # No parents
    # (no ancestors)


    # 2. Break into two parts
    # Has parent1 or parent2
    # Count ancestors of each one
    # (plus parent1, parent2 themselves)



    # 3. Combine the result
```



Recursion and Objects

```
def num_ancestors(p):
    """Returns: num of known ancestors
    Pre: p is a Person"""
    # 1. Handle base case.
    if p.parent1 == None and p.parent2 == None:
        return 0


    # 2. Break into two parts
    parent1s = 0
    if p.parent1 != None:
        parent1s = 1+num_ancestors(p.parent1)
    parent2s = 0
    if p.parent2 != None:
        parent2s = 1+num_ancestors(p.parent2)

    # 3. Combine the result
    return parent1s+parent2s
```



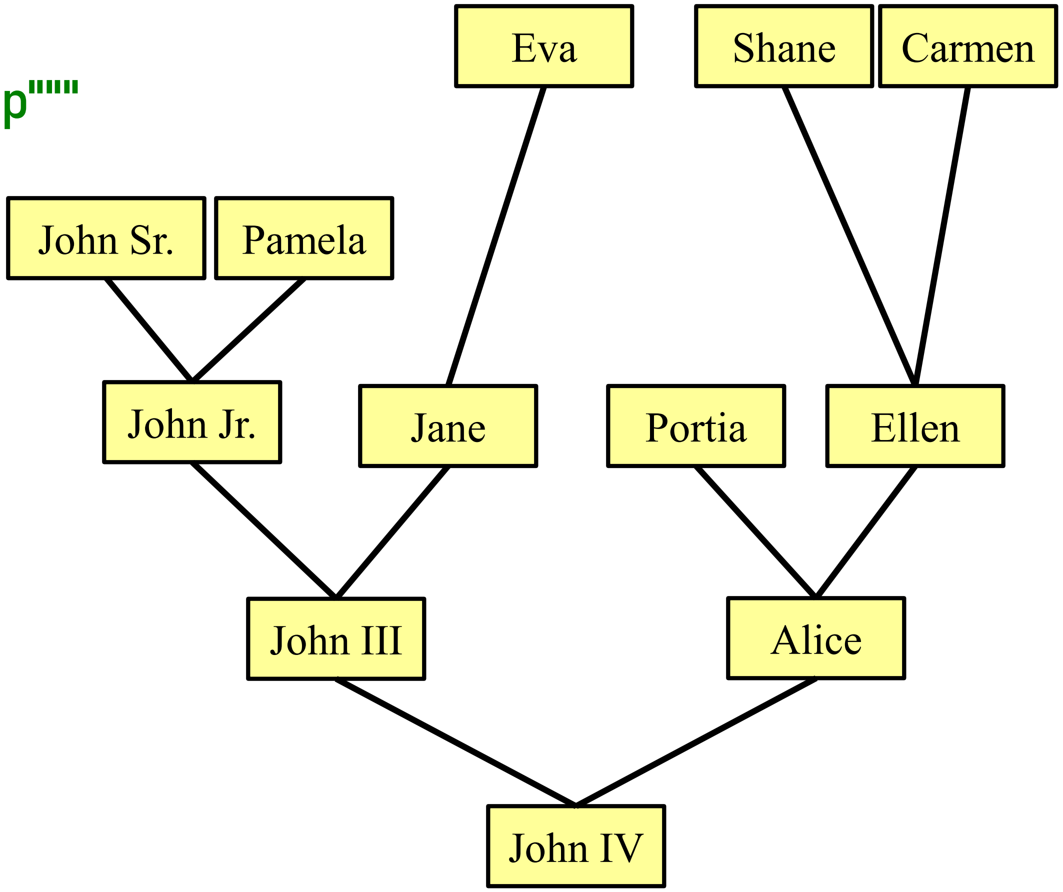
Recursion and Objects

```
def num_ancestors(p):
    """Returns: num of known ancestors
    Pre: p is a Person"""
    # 1. Handle base case.
    if p.parent1 == None and p.parent2 == None:
        return 0

    # 2. Break into two parts
    parent1s = 0
    if p.parent1 != None:
        parent1s = 1+num_ancestors(p.parent1s)
    parent2s = 0
    if p.parent2 != None:
        parent2s = 1+num_ancestors(p.parent2s)

    # 3. Combine the result
    return parent1s+parent2s
```

We don't actually need this.
It is handled by the conditionals in #2.

Exercise: All Ancestors

```
def all_ancestors(p):
    """Returns: list of all ancestors of p"""
    # 1. Handle base case.
    # 2. Break into parts.
    # 3. Combine answer.
```



Optional practice question. Try it after you complete this week’s lab exercise.