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Lecture 26:
More on Algorithms for Sorting

CS 1110

Introduction to Computing Using Python

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Announcements

- Discussion sections this week
  - First 10 minutes dedicated to getting your started on A6
  - Remaining time is office hour for your A6/Prelim 2 questions
- Final Exam on May 21st 1:30-4pm. Your assigned exam session (in-person or online) is shown in CMS. *Submit a "**regrade request**" in CMS by May 12* if you have a *legitimate* reason for requesting a change. If you have an exceptional circumstance for switching from in-person to online, you must upload to CMS your college's approval of your modality change.

More Announcements

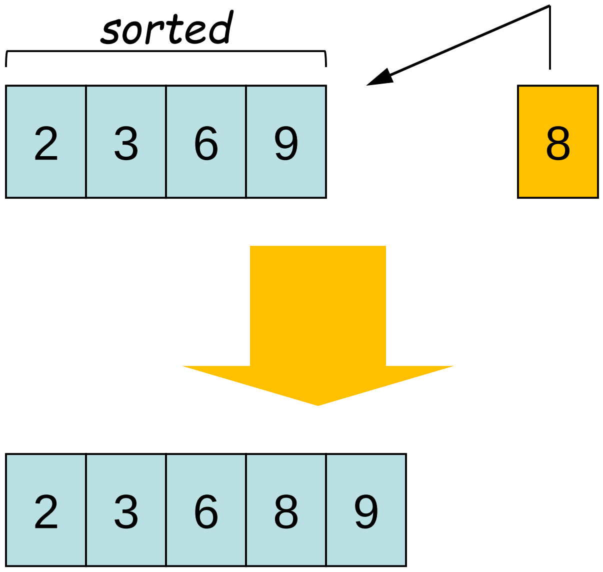
- A6 due on Friday
  - Remember academic integrity
- Expected release dates of solutions and feedback
  - A5 solutions: Wed May 12
  - A4 grades and feedback: Thurs May 13
  - A6 solutions: Tues May 18
  - A5 grades and feedback: Thurs May 20
  - Final exam grades and feedback: Tues May 25
  - A6 grades and feedback: Fri May 28

Algorithms for Sorting

- Well known algorithms
  - focus on reviewing programming constructs (`while` loop) and analysis
  - will not use built-in methods such as **`sort`**, **`index`**, **`insert`**, etc.
- Today we'll discuss merge sort and compare it to insertion sort, which we discussed last lecture
- More on the topic in next course, CS 2110!

The Insertion Process of Insertion Sort

- Given a sorted list x, insert a number y such that the result is sorted
- Sorted: arranged in ascending (small to big) order



We'll call this process a "push down," as in push a value down until it is in its sorted position

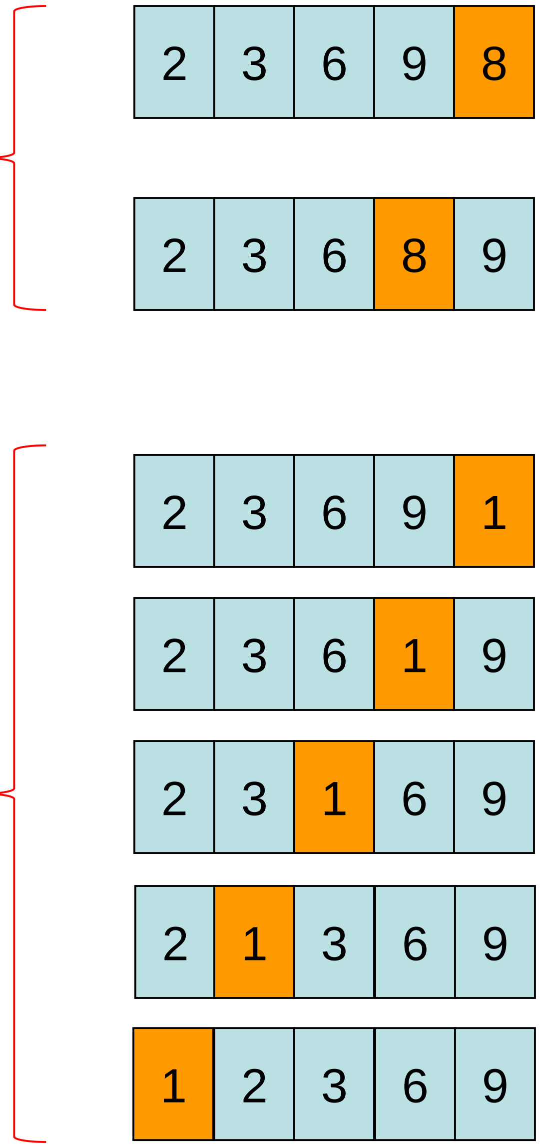
Algorithm Complexity

- Count the number of comparisons needed
- In the worst case, need i comparisons to push down an element in a sorted segment with i elements.

How much work is a push down?

push down a "big" value

push down a "small" value



This push down takes 2 comparisons

This push down takes 4 comparisons. Worst case scenario: ***n*** comparisons needed to push down into a length ***n*** sorted segment.

Algorithm Complexity (Q)

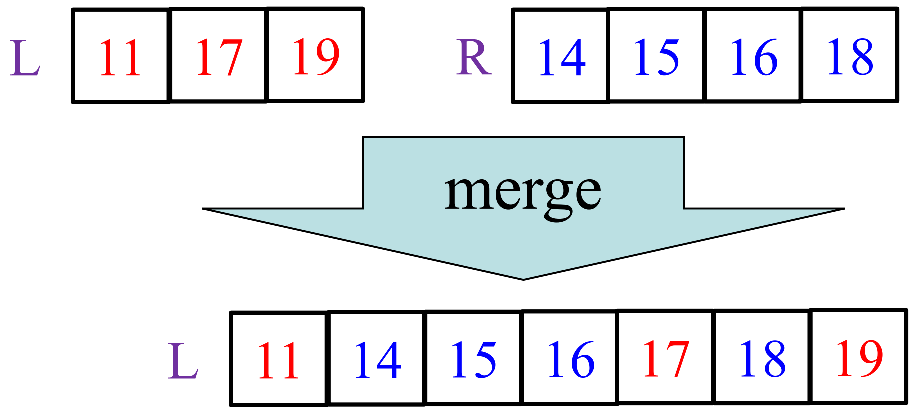
```
def swap(b, h, k):
    ⋮

def push_down(b, k):
    while k > 0 and b[k-1] > b[k]:
        swap(b, k-1, k)
        k= k-1

def insertion_sort(b):
    for i in range(1,len(b)):
        push_down(b, i)
```

Count (approximately) the number of comparisons needed to sort a list of length n

A. ~ 1 comparison
B. ~ n comparisons
C. ~ n^2 comparisons
D. ~ n^3 comparisons
E. I don't know

Which algorithm does Python's `sort` use?

- Recursive algorithm that runs much faster than insertion sort for the same size list (when the size is big)!
- A variant of an algorithm called "merge sort"
- Based on the idea that sorting is hard, but *"merging" two already sorted lists is easy*.



Merge sort: Motivation

Since merging is easier than sorting, if I have two helpers, I'd…

- Give each helper half the array to sort
- Then I get back their sorted subarrays and merge them.

What if those two helpers each had two sub-helpers?

And the sub-helpers each had two sub-sub-helpers? And…

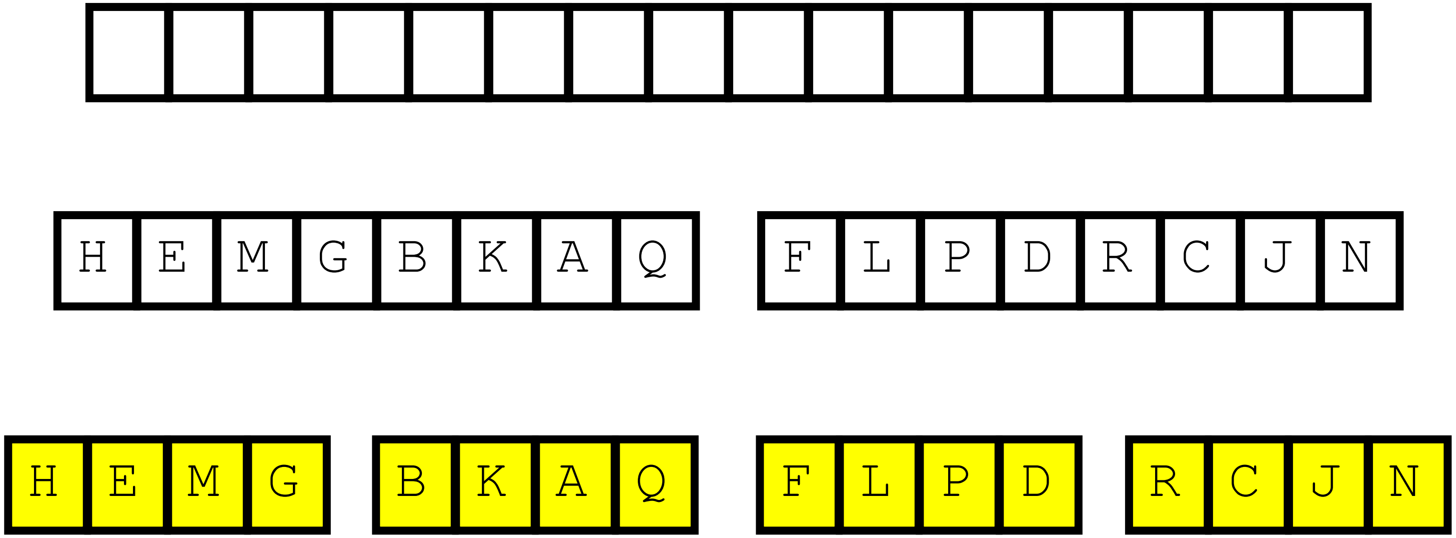
Subdivide the sorting task

H	E	M	G	B	K	A	Q	F	L	P	D	R	C	J	N

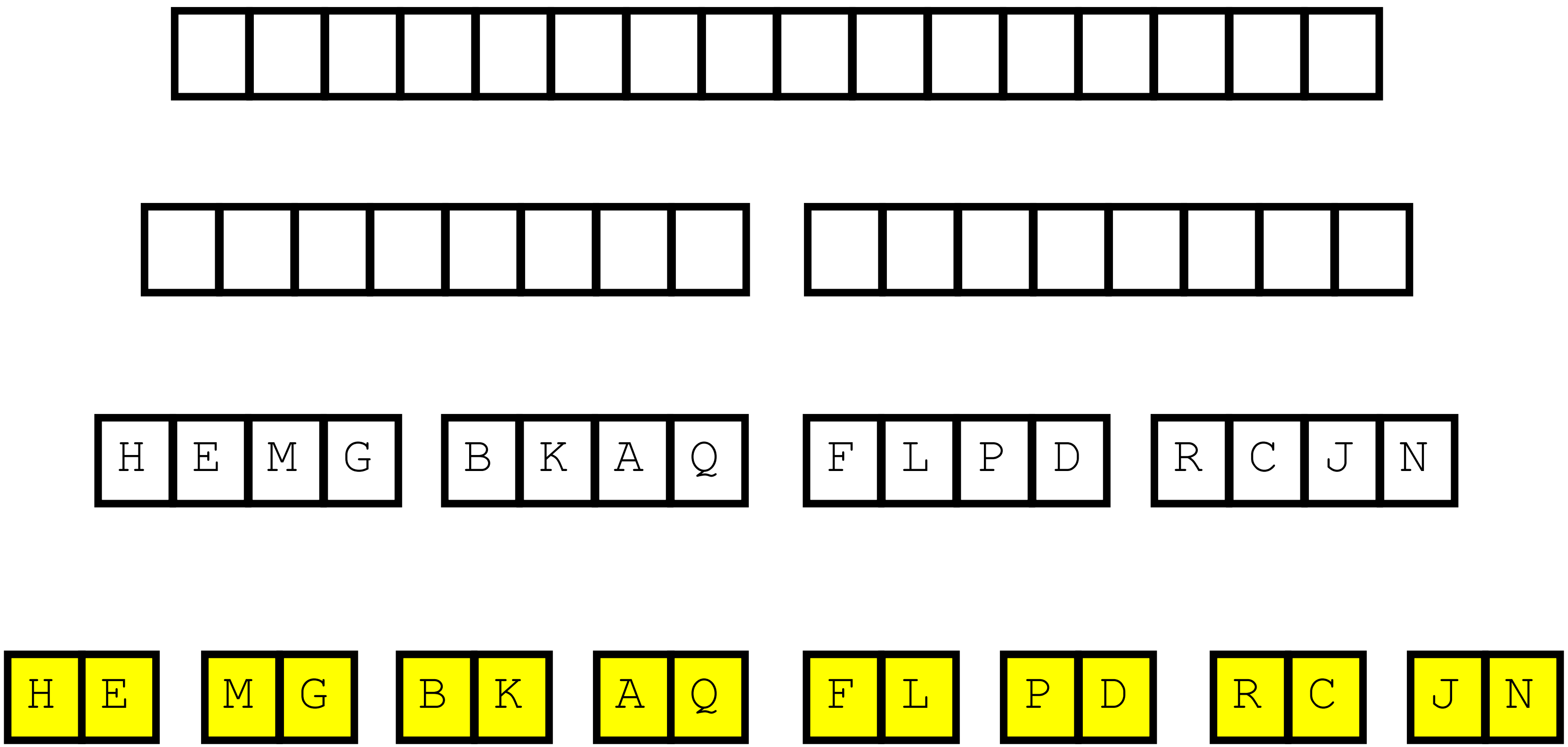
H	E	M	G	B	K	A	Q

F	L	P	D	R	C	J	N

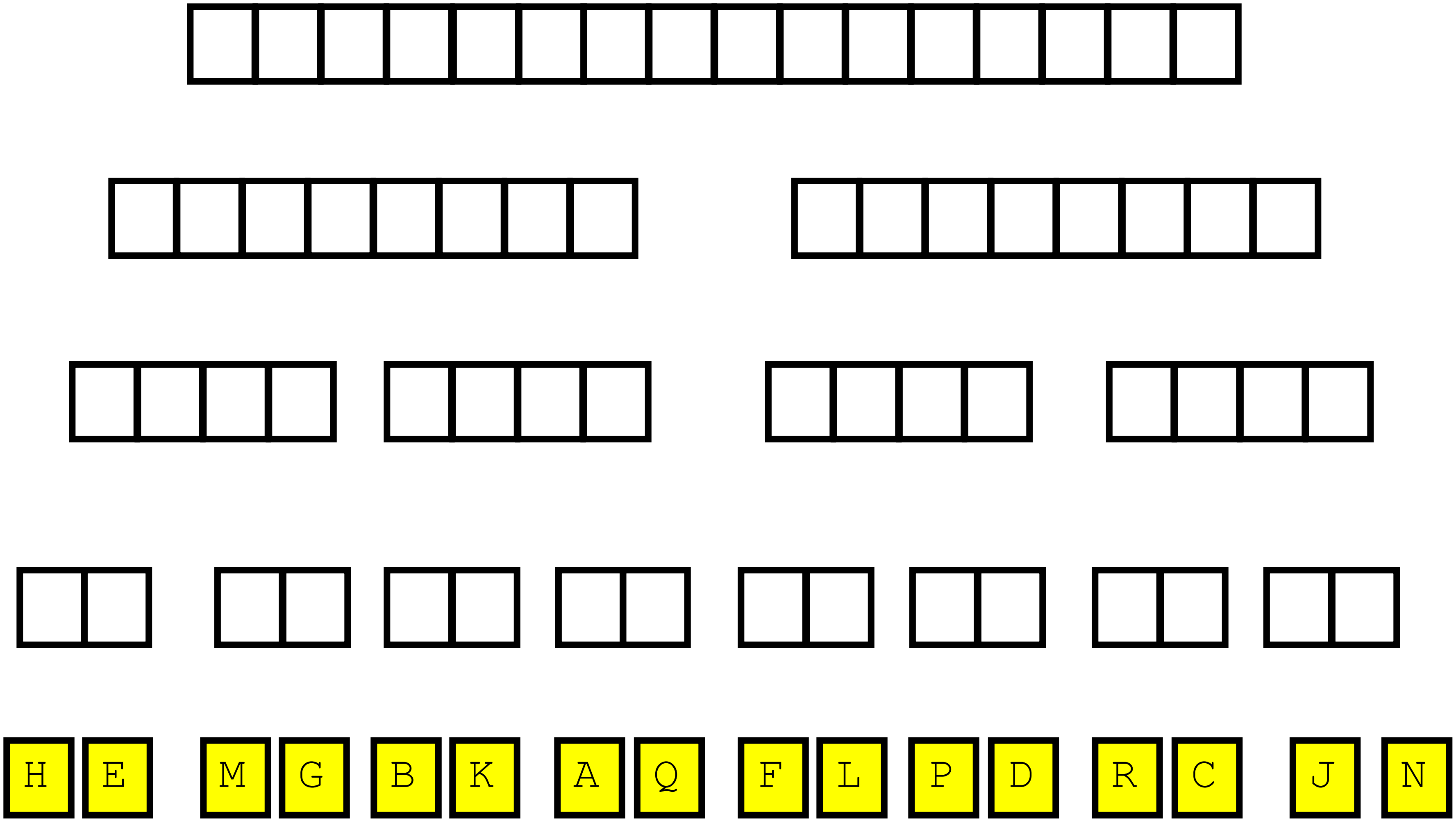
Subdivide again



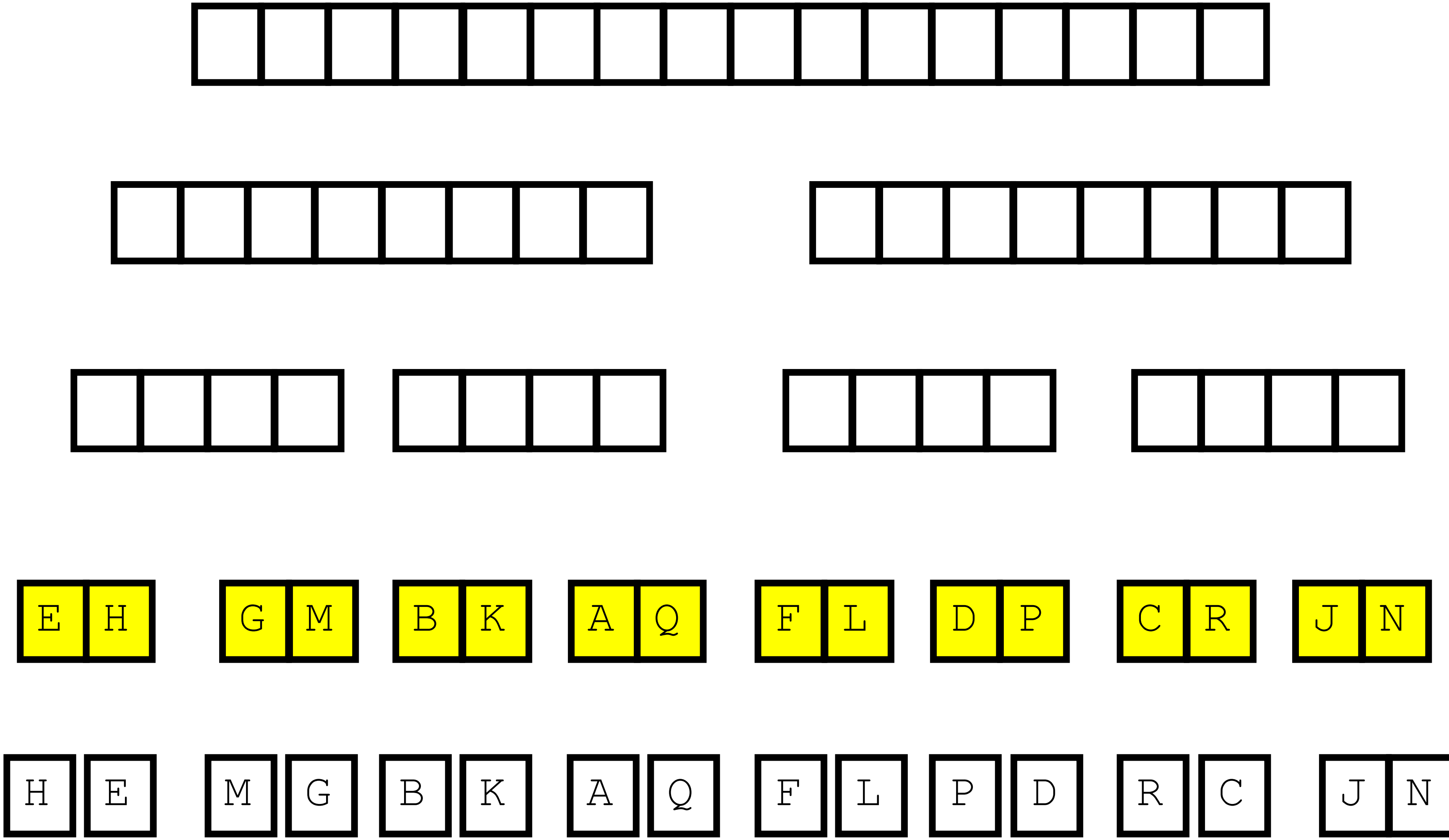
And again



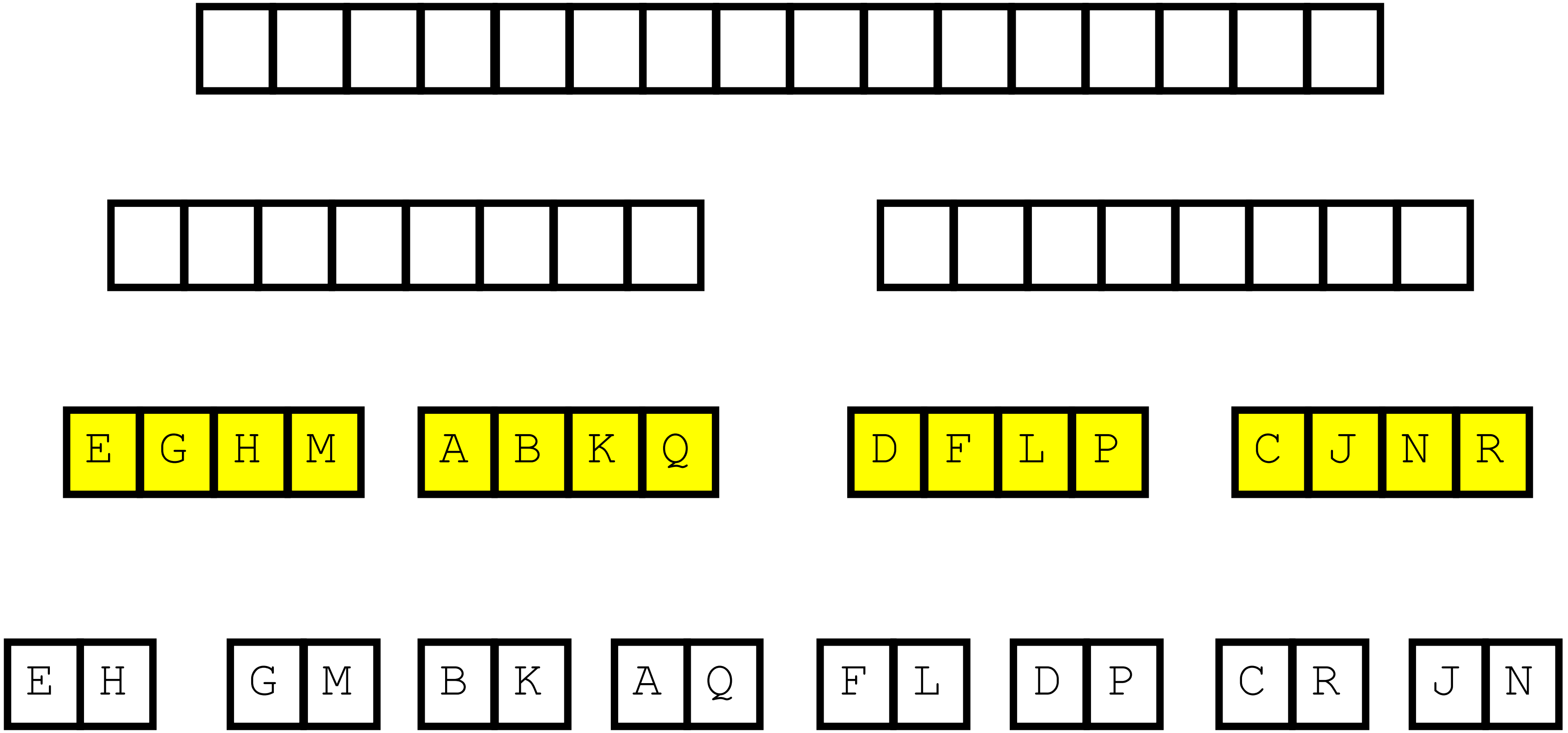
And one last time



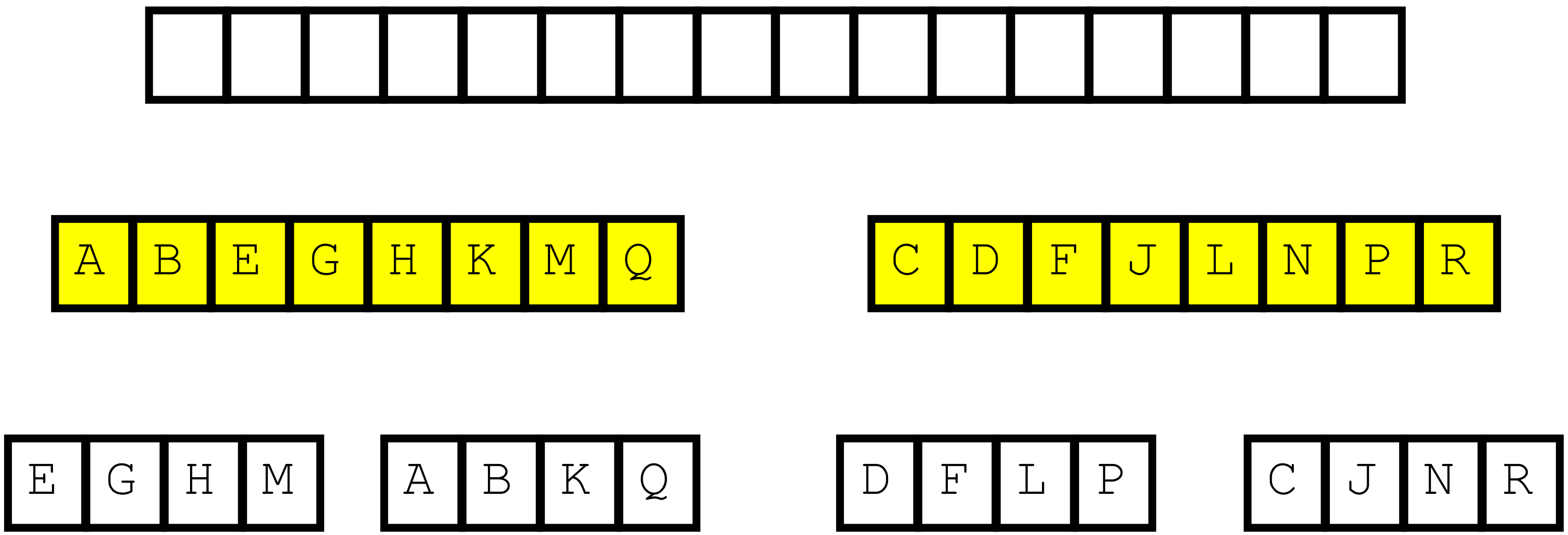
Now merge



And merge again



And again



And one last time

A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R

A	B	E	G	H	K	M	Q

C	D	F	J	L	N	P	R

Done!

A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R

```python
def mergeSort(li):
    """Sort list li using Merge Sort"""
    if len(li) > 1:
        # Divide into two parts
        mid= len(li)//2
        left= li[:mid]
        right= li[mid:]

        # Recursive calls
        mergeSort(left)
        mergeSort(right)

        # Merge left & right back to li
        …
```

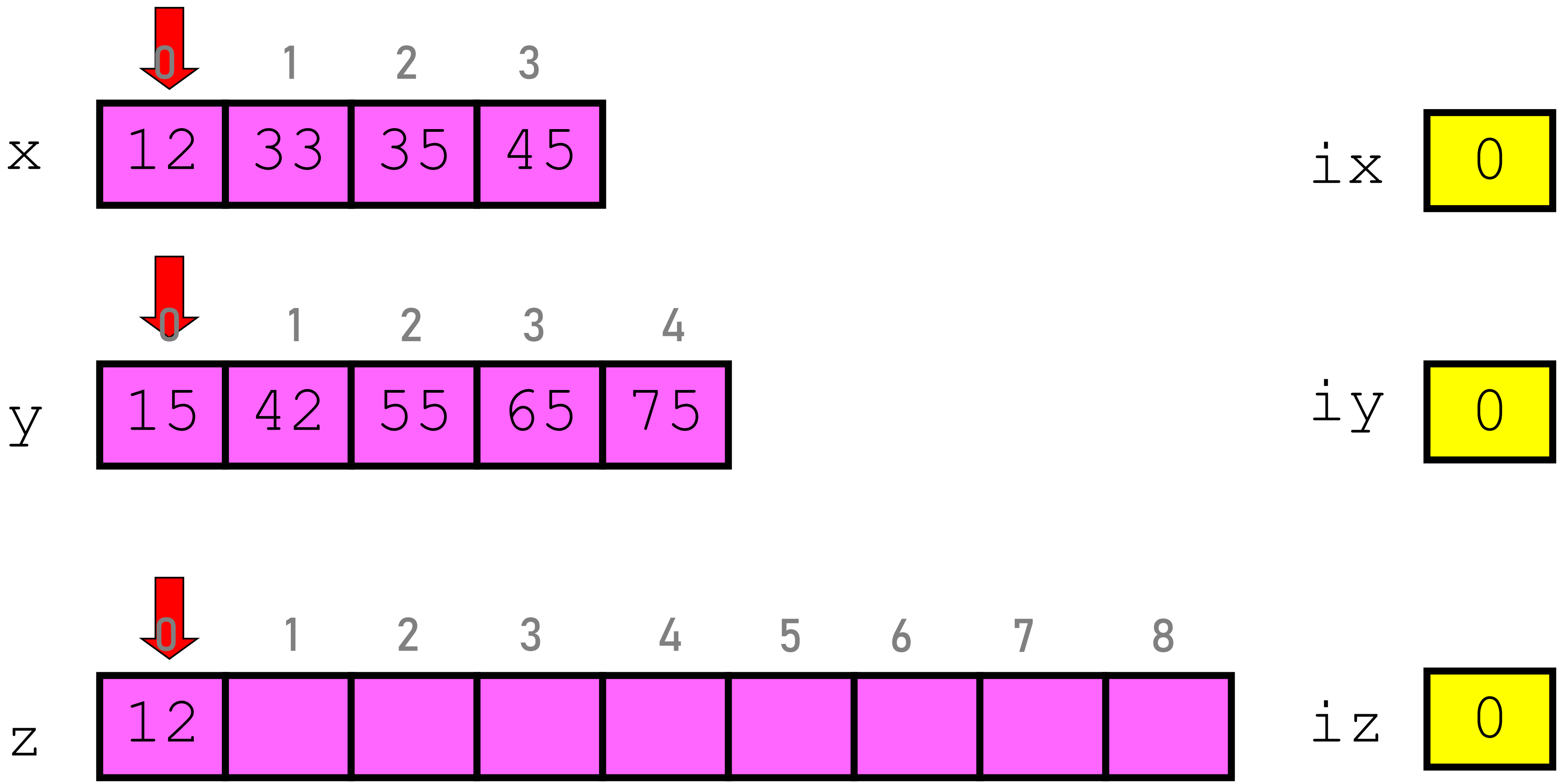
The central sub-problem is the merging of two sorted lists into one single sorted list

12	33	35	45

15	42	55	65	75

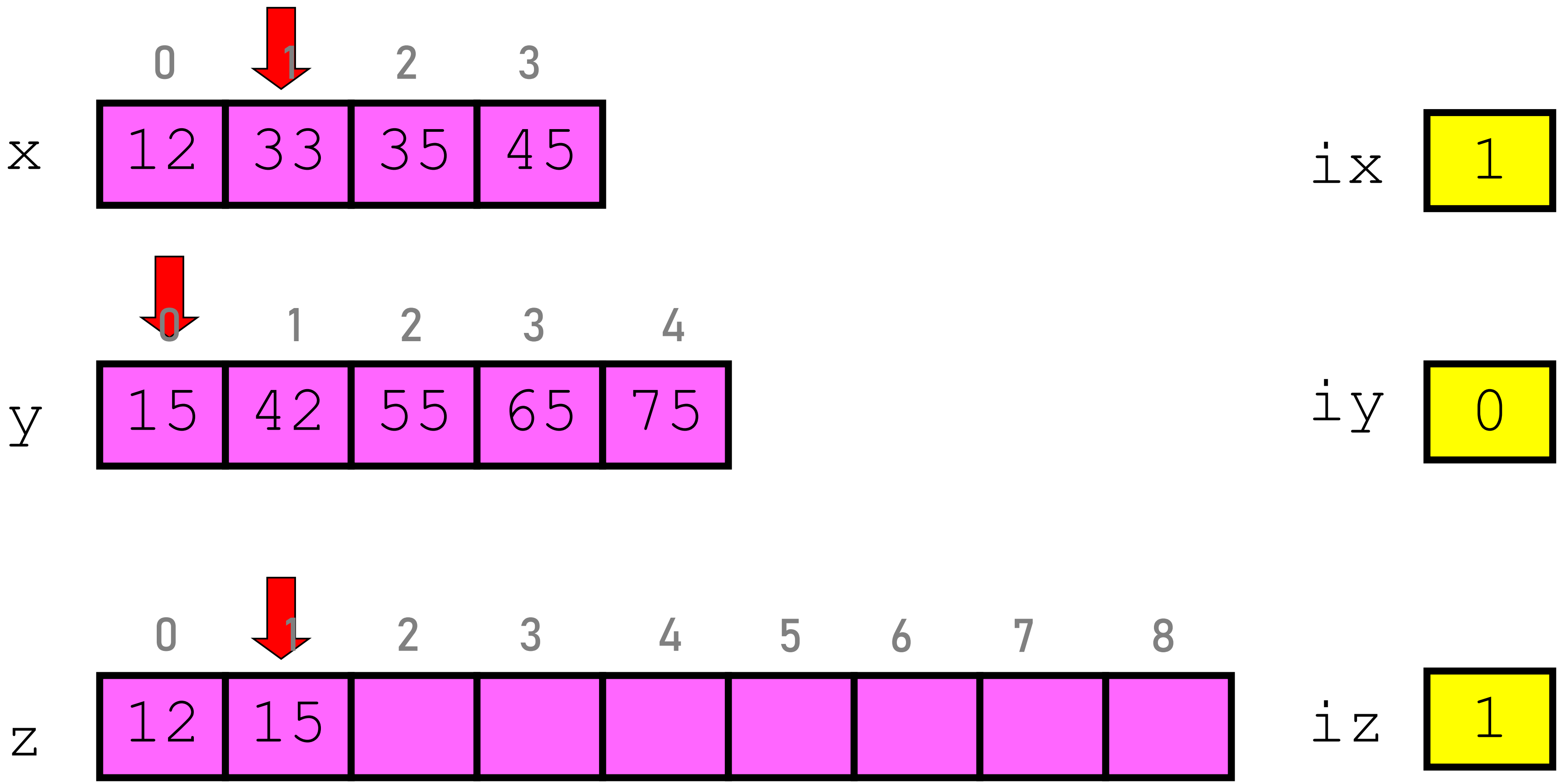
12	15	33	35	42	45	55	65	75

Merge



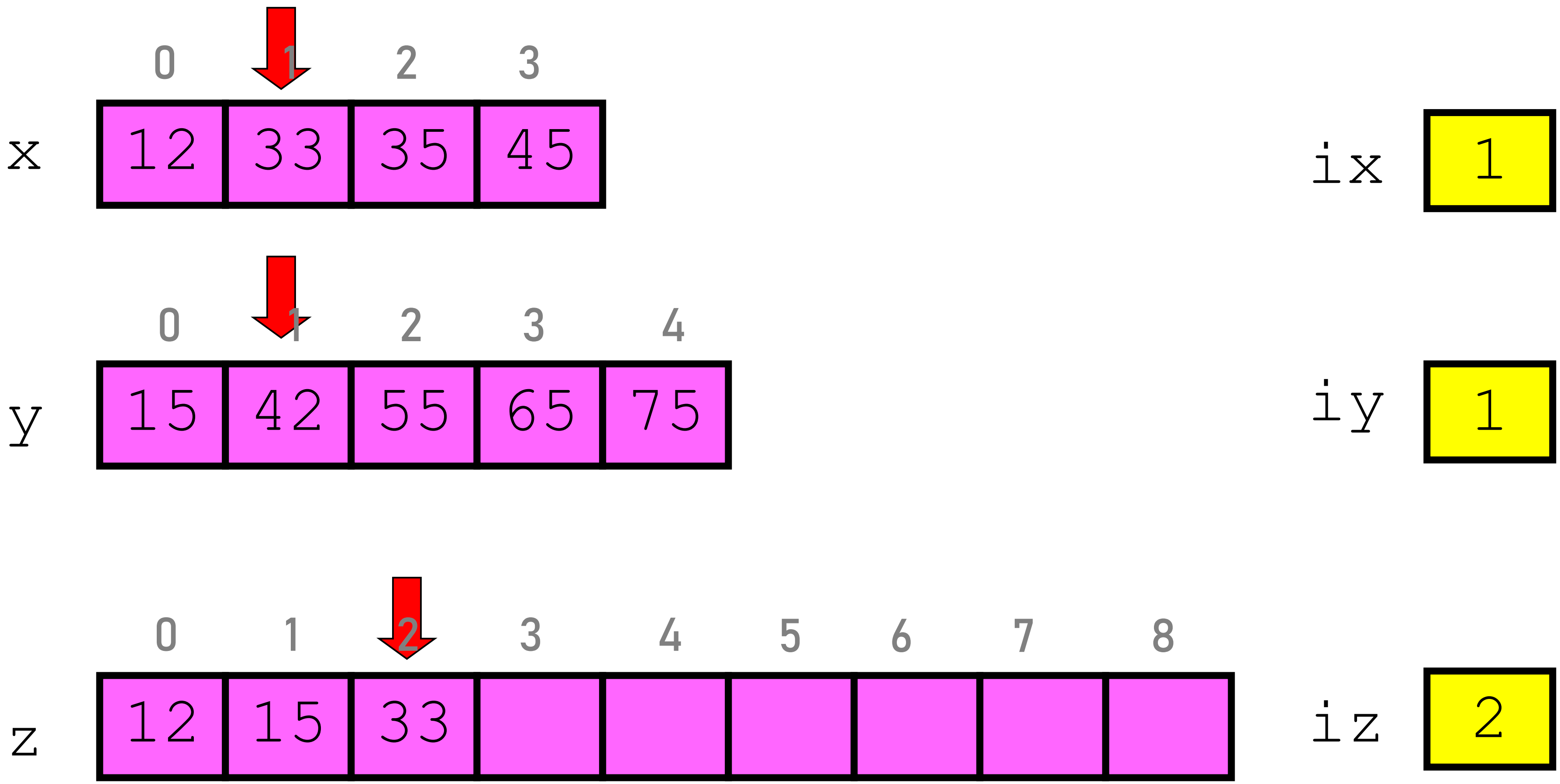
ix<4 and iy<5 → x(ix) <= y(iy) YES

Merge



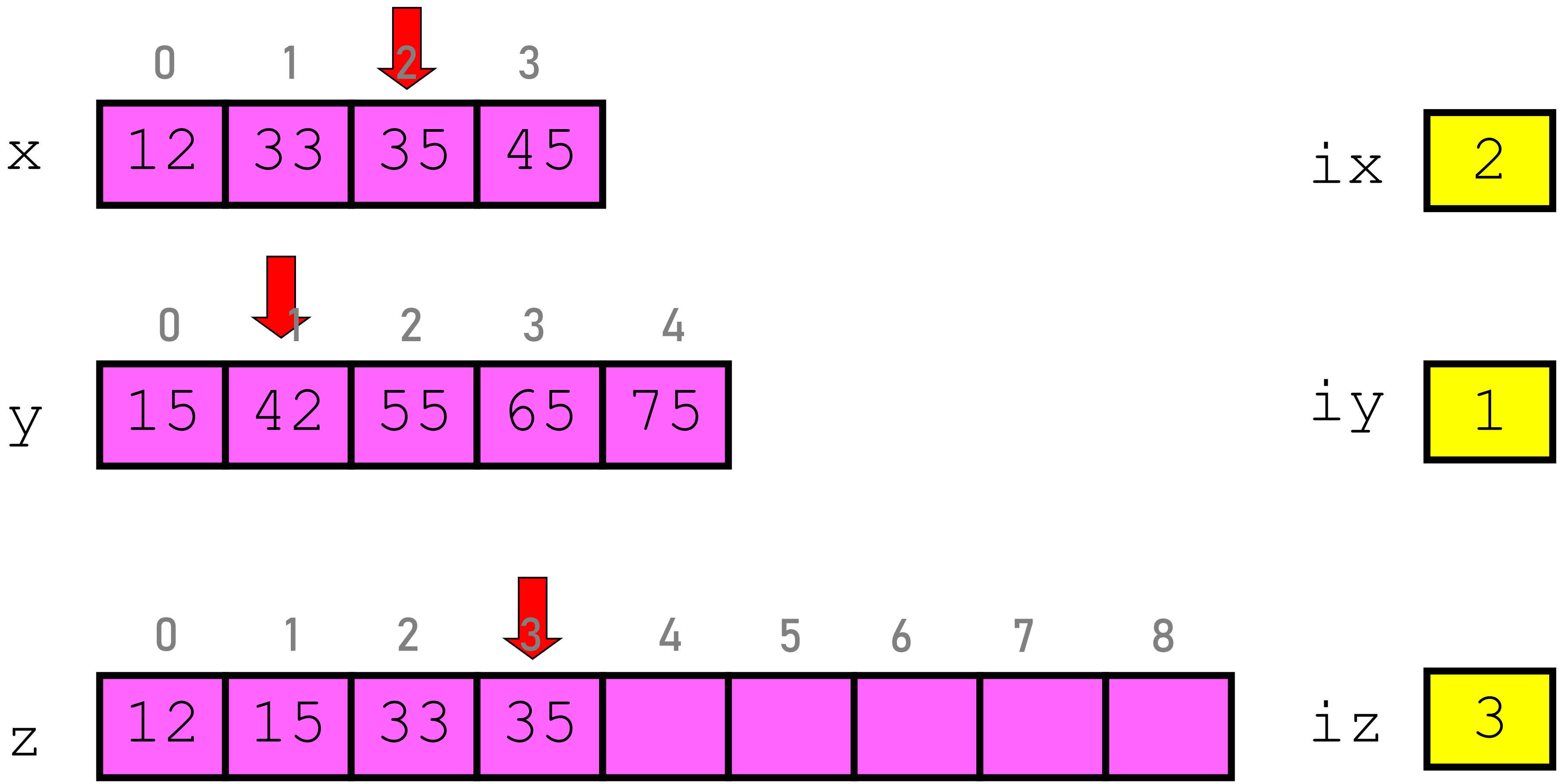
ix<4 and iy<5 → x(ix) <= y(iy) NO

Merge



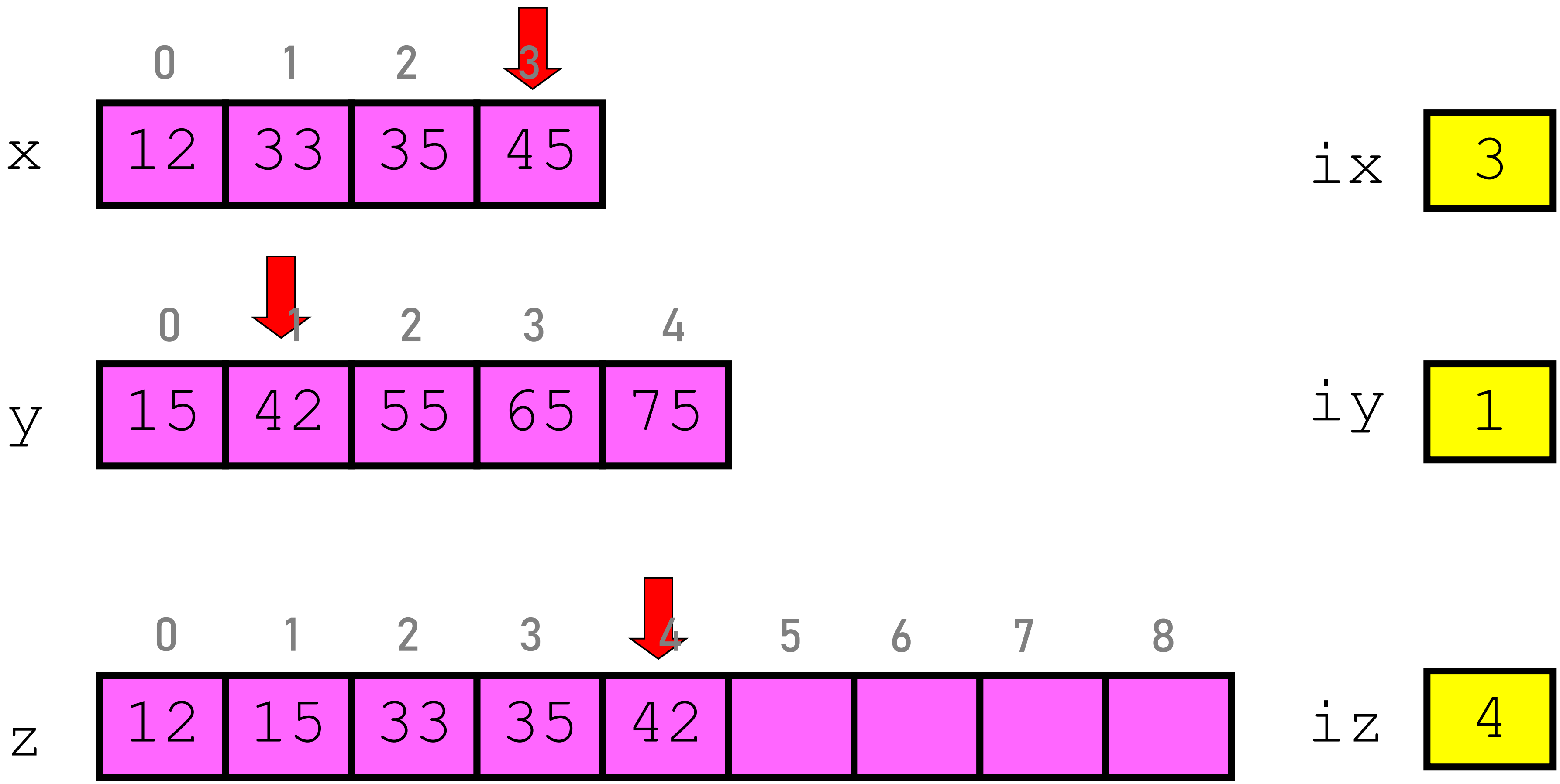
ix<4 and iy<5 → x(ix) <= y(iy) YES

Merge



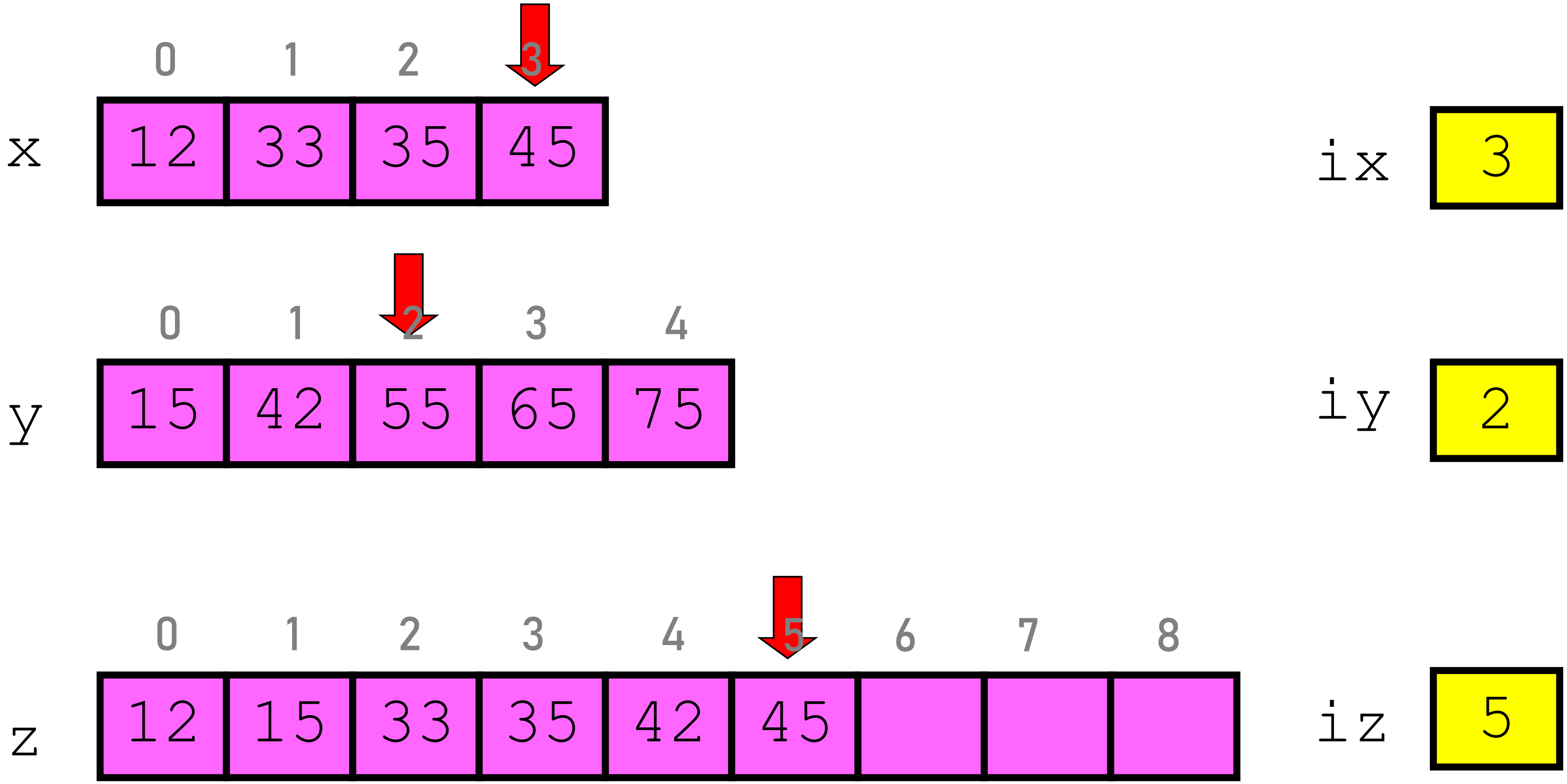
ix<4 and iy<5 → x(ix) <= y(iy) YES

Merge



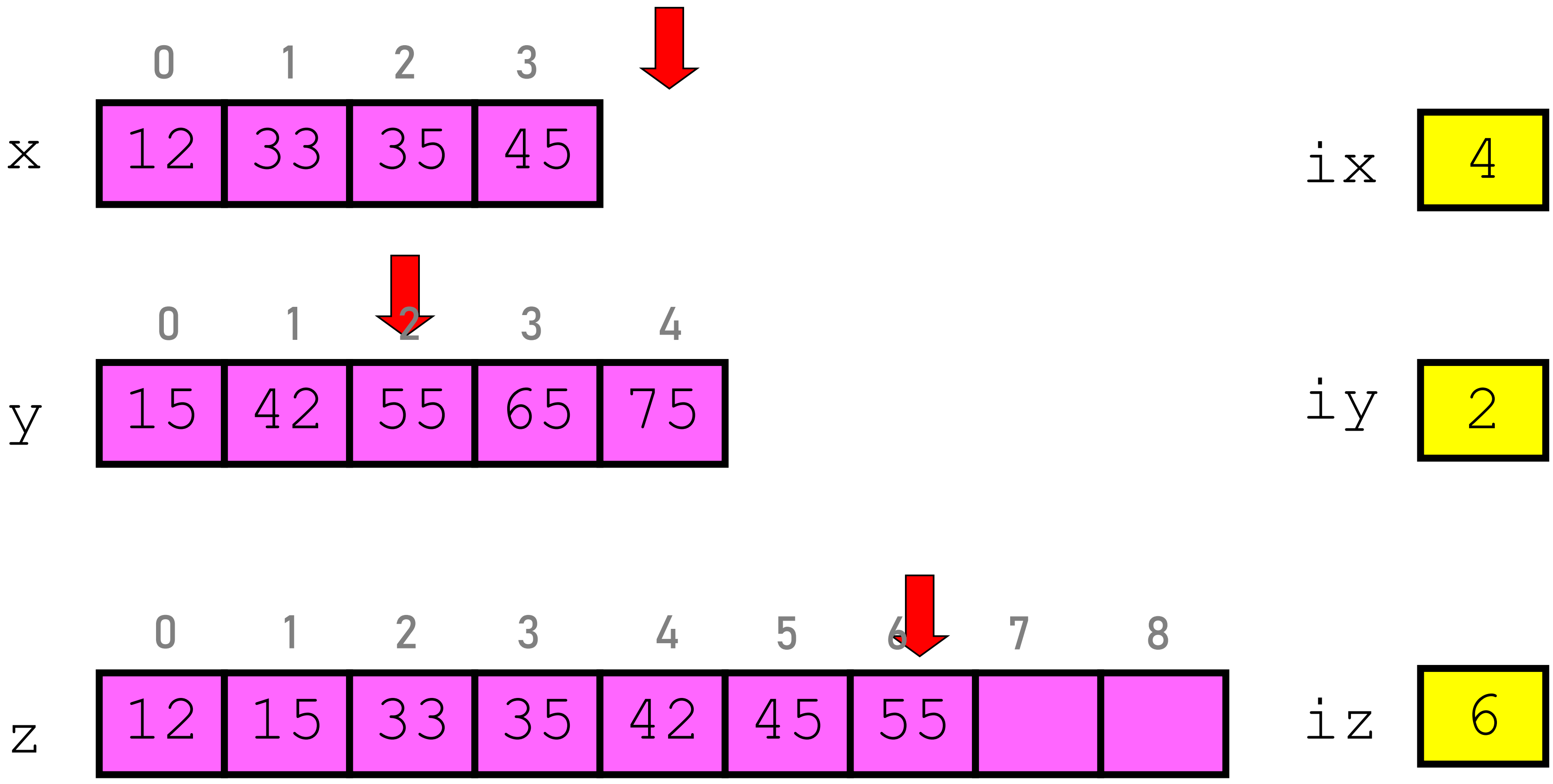
`ix<4 and iy<5` → `x(ix) <= y(iy)` NO

Merge



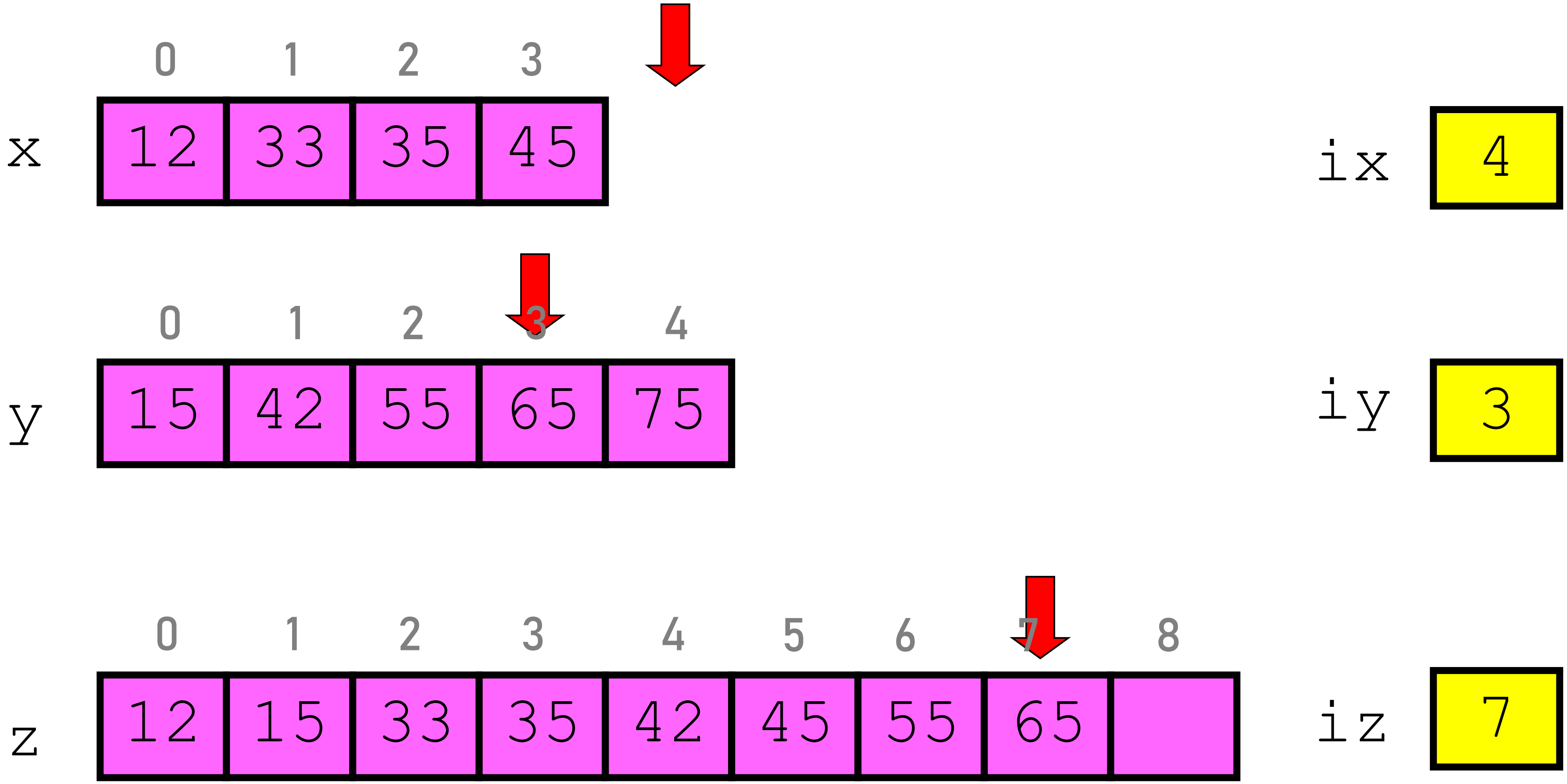
ix<4 and iy<5 → x(ix) <= y(iy) YES

Merge



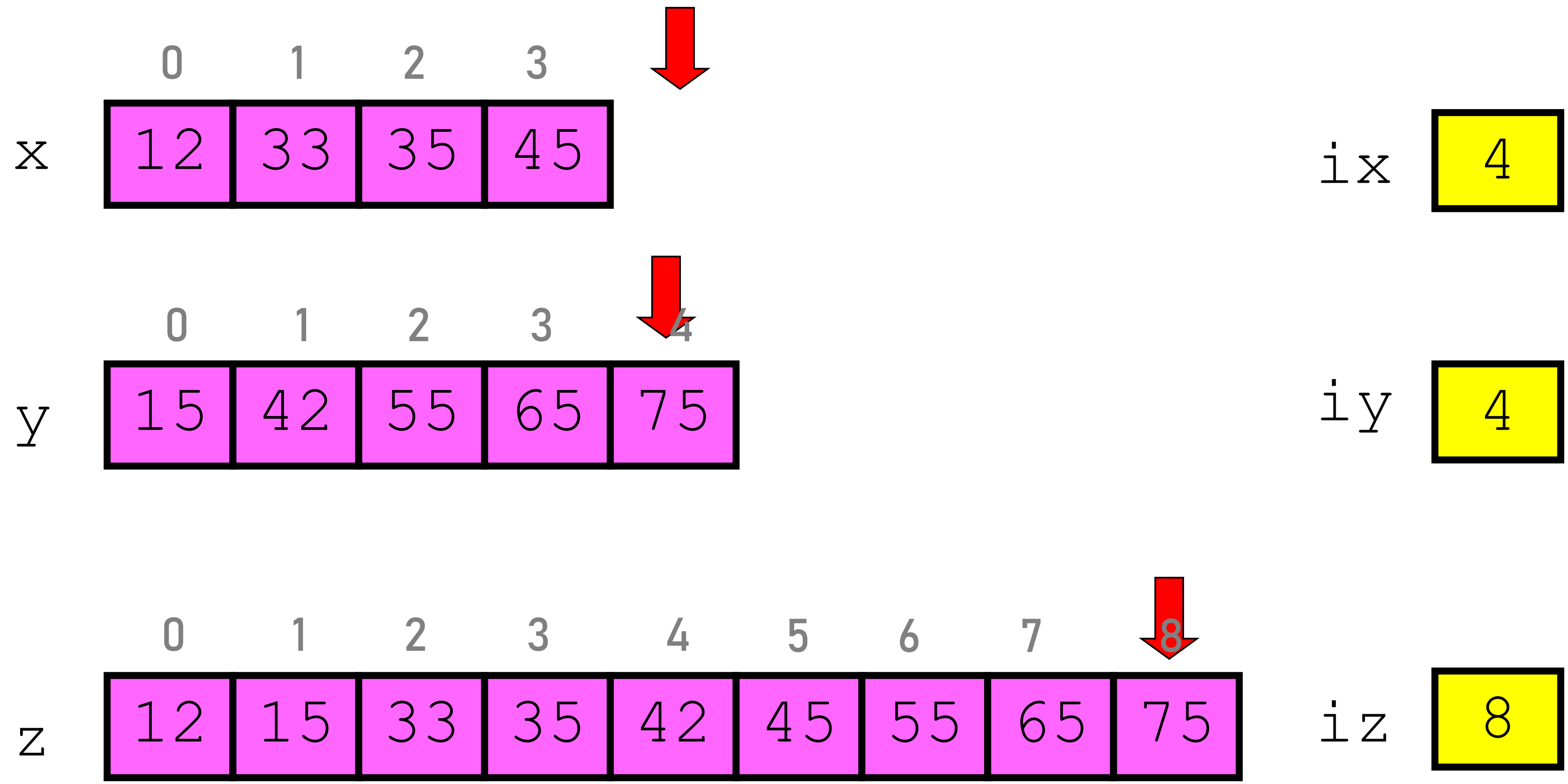
ix at 4 → take y(iy)

Merge



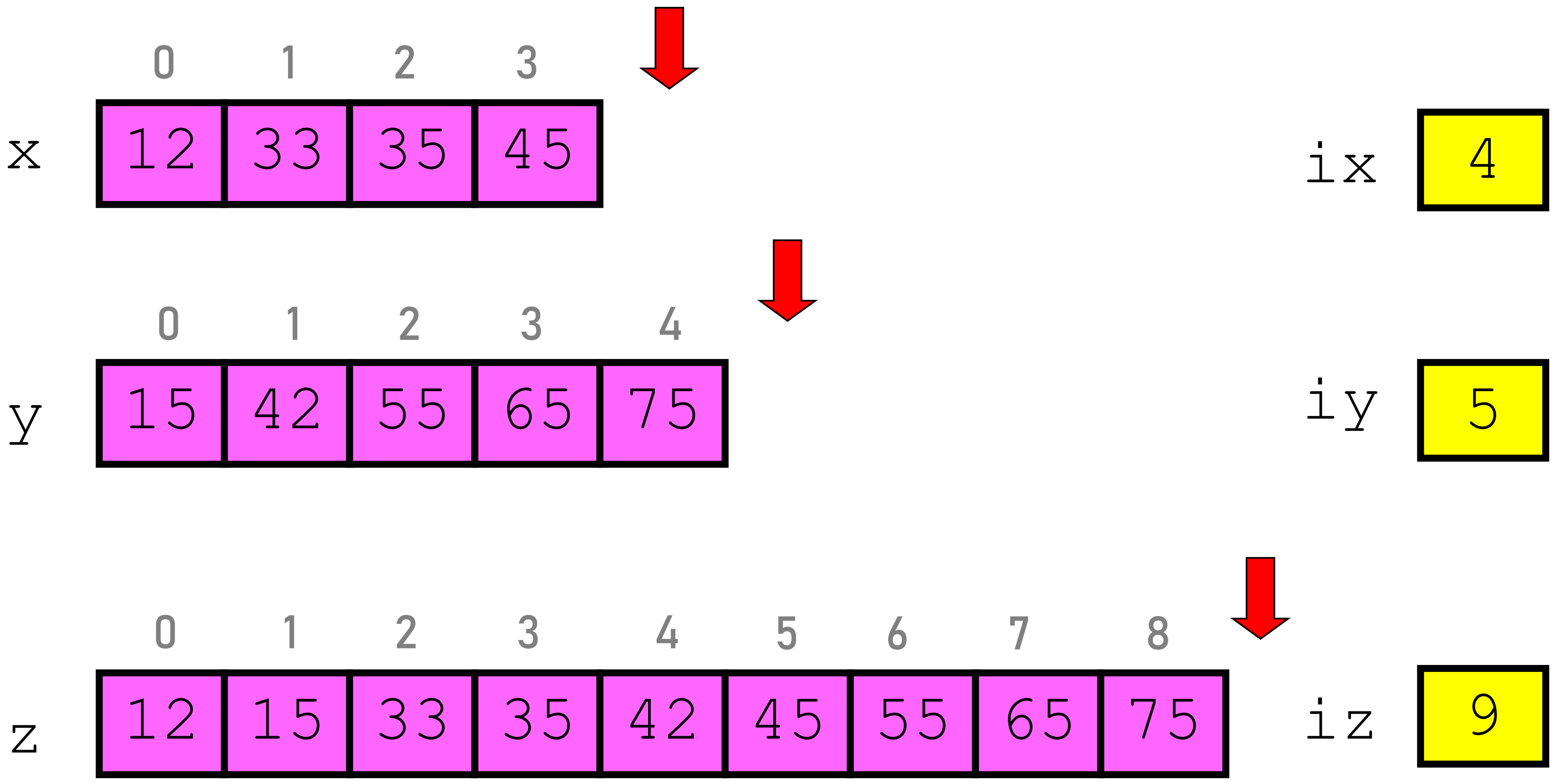
iy < 5 → take y(iy)

Merge



`iy < 5` → `take y(iy)`

Merge



0
1
2
3
x
12
33
35
45
ix
4
0
1
2
3
4
y
15
42
55
65
75
iy
5
0
1
2
3
4
5
6
7
8
z
12
15
33
35
42
45
55
65
75
iz
9

```
# Given lists x and y and list z, which has
# the combined length of x and y...
nx = len(x); ny = len(y)

ix = 0; iy = 0; iz = 0;
while ix<nx and iy<ny
    if  x[ix] <= y[iy]:
        z[iz]= x[ix];  ix=ix+1
    else:
        z[iz]= y[iy];  iy=iy+1
    iz=iz+1

while ix<nx  # copy any remaining x-values
  z[iz]= x[ix];  ix=ix+1;  iz=iz+1

while iy<ny  # copy any remaining y-values
  z[iz]= y[iy];  iy=iy+1;  iz=iz+1
```

How do merge sort and insertion sort compare?

- Insertion sort: (worst case) makes i comparisons to insert an element in a sorted array of i elements. For an array of length n:

  ____________________ for big n

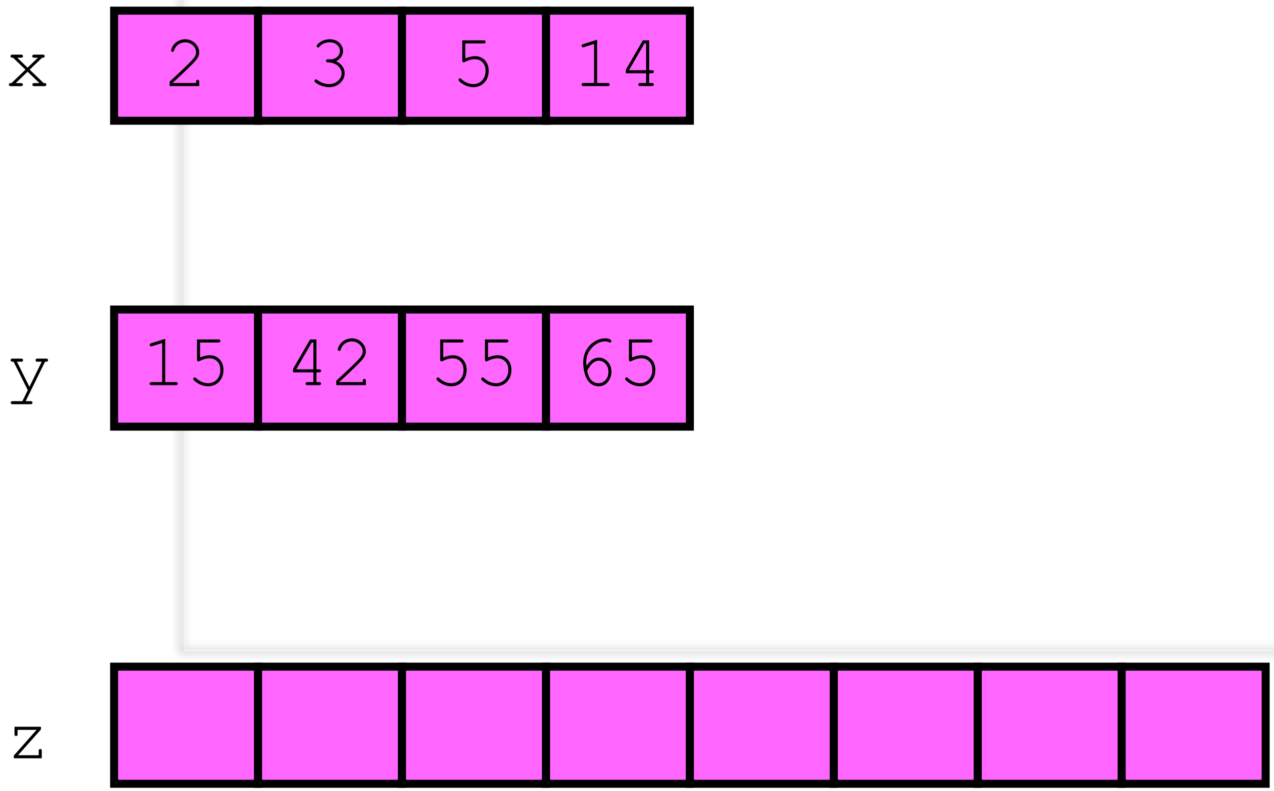
- Merge sort: ____________________

```
def mergeSort(li):
    """Sort list li using Merge Sort"""
    if len(li) > 1:
        # Divide into two parts
        mid= len(li)/2
        left= li[:mid]
        right= li[mid:]

        # Recursive calls
        mergeSort(left)
        mergeSort(right)

        # Merge left & right back to li
        …
```

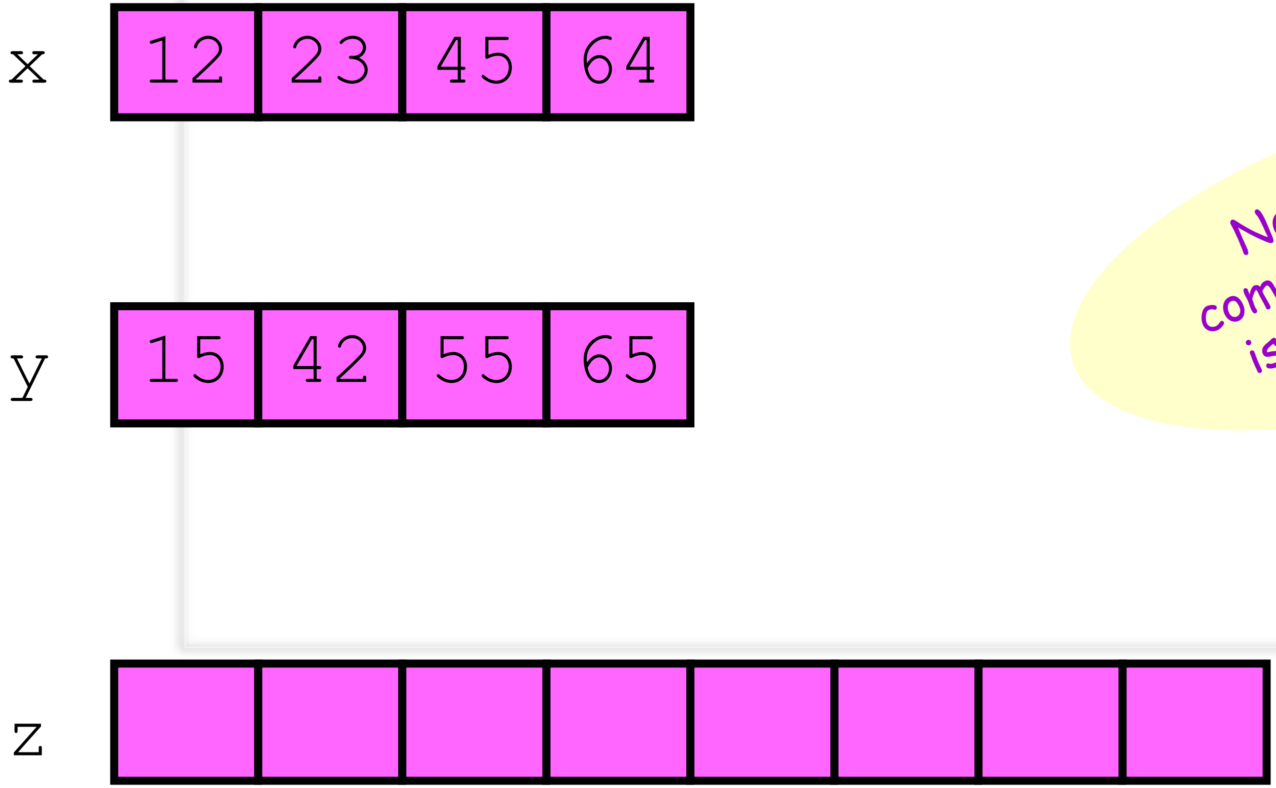
All the comparisons between list elements are done during merge

```
# Given lists x and y and list z, which has
# the combined length of x and y...
nx = len(x); ny = len(y)

ix = 0; iy = 0; iz = 0;
while ix<nx and iy<ny
    if x[ix] <= y[iy]:
        z[iz]= x[ix];  ix=ix+1
    else:
        z[iz]= y[iy];  iy=iy+1
    iz=iz+1

while ix<nx  # copy any remaining x-values
  z[iz]= x[ix];  ix=ix+1;  iz=iz+1

while iy<ny  # copy any remaining y-values
  z[iz]= y[iy];  iy=iy+1;  iz=iz+1
```

Merge – best case scenario

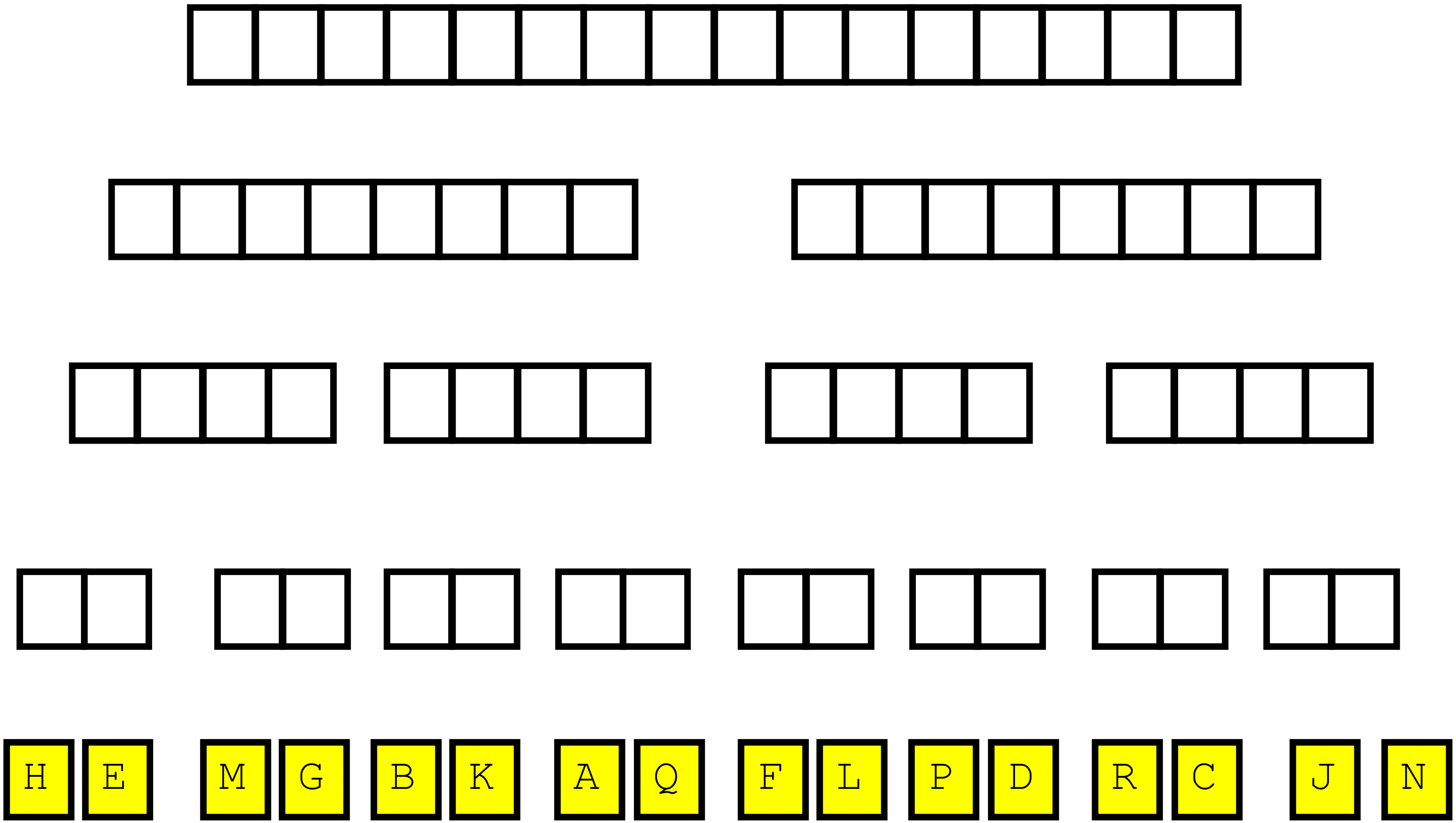


Merge – worst case scenario



Need to do n-1 comparisons where n is total number of elements merged

Merge sort: about $\log_2(n)$ "levels"; about n comparisons each level



How do merge sort and insertion sort compare?

- Insertion sort: (worst case) makes i comparisons to insert an element in a sorted array of i elements. For an array of length n:

  $1+2+\ldots+(n-1) = n(n-1)/2$, say n^2 for big n

- Merge sort: $n \cdot \log_2(n)$ comparisons

Order of magnitude difference

- Should we always use merge sort then? Python actually uses a variant that combines merge sort and insertion sort!