26 Mar 2021

Airline scheduling (§7.9)
Project selection (§7.11)

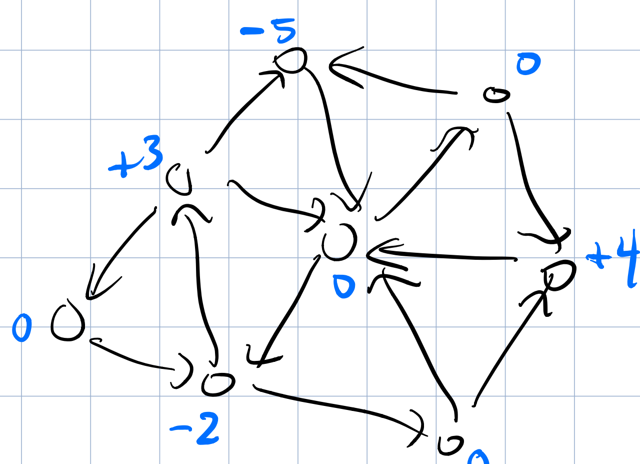
Announcements:

(1) Midterm TA evaluations due Monday.
(2) Problem Set 6 due Thurs. night.

Networks with circulations and demands:

- generalizes max flow by allowing imbalance $d(v) = \sum_{e \text{ into } v} f(e) - \sum_{e \text{ out of } v} f(e)$ to be non-zero at vertices other than s,t.
- DECISION PROBLEM: given $d(v)$ for each v does there exist circulation with that set of imbalances?

E.g.



Circulation with demands and lower bounds:

- Each vertex specifies $d(v)$ which could be pos. or neg.
- Each edge specifies an interval $[\ell(e), c(e)]$

Decision problem: does there exist flow $f(e)$ s.t.

1. [Modified capacity constraint]

$$\ell(e) \le f(e) \le c(e) \quad \forall e \in E$$

2. [Modified conservation constraint]

$$d(v) = \sum_{e \text{ into } v} f(e) - \sum_{e \text{ out of } v} f(e) \quad \forall v \in V$$

Reducing this problem to ordinary circulation:
write $f(e)$ as $\ell(e) + g(e)$.
The two constraints become:

$$\text{(1')} \quad 0 \le g(e) \le \underbrace{\boxed{c(e) - \ell(e)}}_{c'(e)} \quad \forall e \in E$$

$$\text{(2')} \quad \underbrace{\boxed{d(v) - \sum_{e \text{ into } v} \ell(e) + \sum_{e \text{ out of } v} \ell(e)}}_{d'(v)} = \sum_{e \text{ into } v} g(e) - \sum_{e \text{ out of } v} g(e) \quad \forall v \in V$$

(A) circulation with demands $d(v)$, lower/upper bds $[\ell(e), c(e)]$

$\Longleftrightarrow$ (B) circulation with demands $d'(v)$ capacities $c'(e)$.

To solve (A), compute $d'(v)$, $c'(e)$ $\forall v, e$. Use d', c' as input data to circulation problem. Solve to find $g : E \to \mathbb{R}_{\ge 0}$. Finally if g satisfying constraints is found, let $f(e) = g(e) + \ell(e)$ $\forall e$, that solves (A).

<u>Airline scheduling</u>. An airline promises to operate <u>flight segments</u> each with

- departure airport, departure time
- arrival airport, arrival time.

Operating a flight segment requires

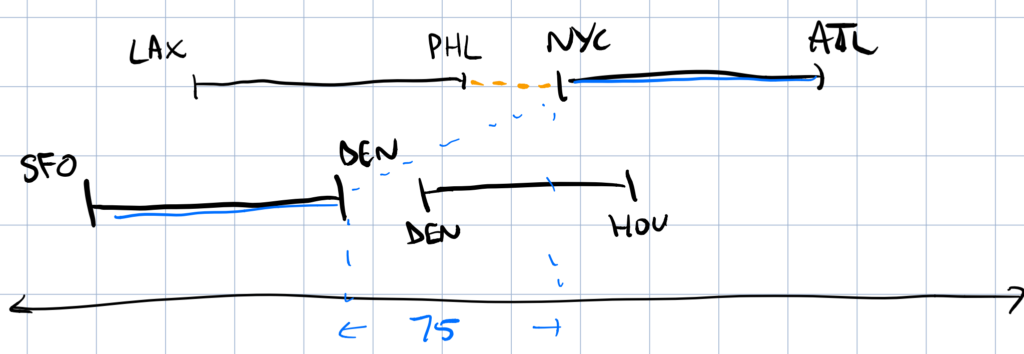
sending airplane from one airport to another at the specified time.

1. An airplane can't be in two places at once.
2. If it flies a segment ending at airport X at time t, and next flight departs from X at t', $t' - t \geq \boxed{30}$.

   time to service the airplane

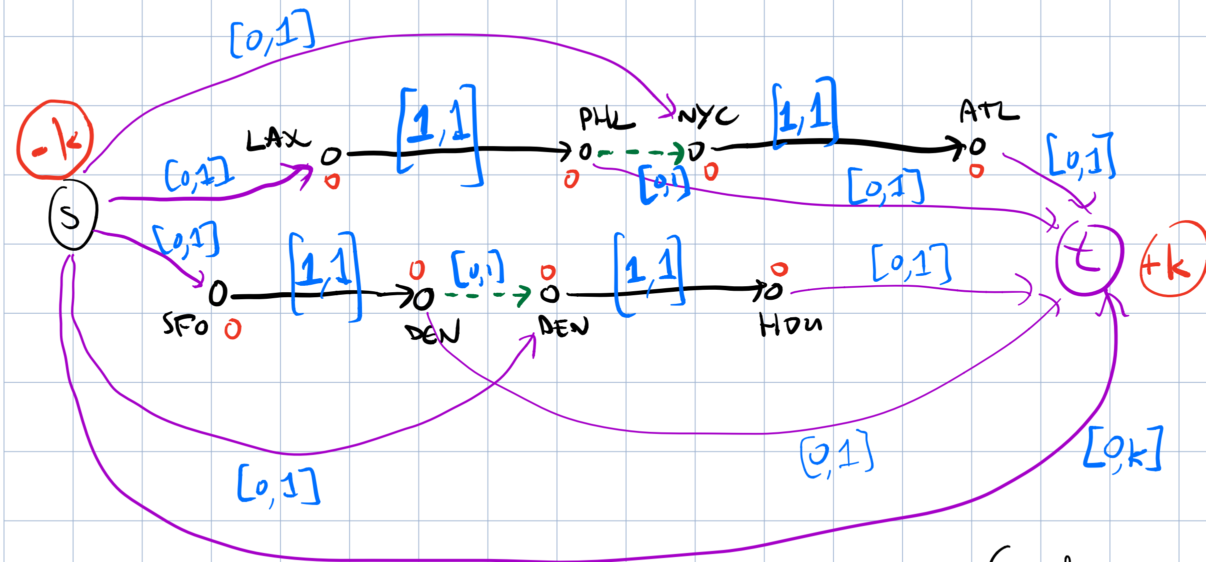
3. If next segment departs Y at t' then $t' - t \geq 30 + d(X,Y)$.

<u>Question:</u> With a fleet of k airplanes can we operate all flight segments?

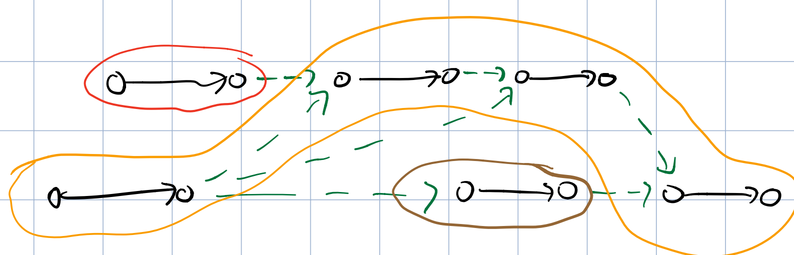


Scheduling one single plane to serve as many flight segments as possible is similar to interval scheduling

This can be modeled as a longest path in a DAG problem and solved with Bellman-Ford.



Greedy may be suboptimal.



Reducing Problems to Min-Cut

Project selection: Set of n projects.

Dependency graph: vertices are projects.

directed edge (u,v) means

"v depends on u. You cannot do v without first doing u"

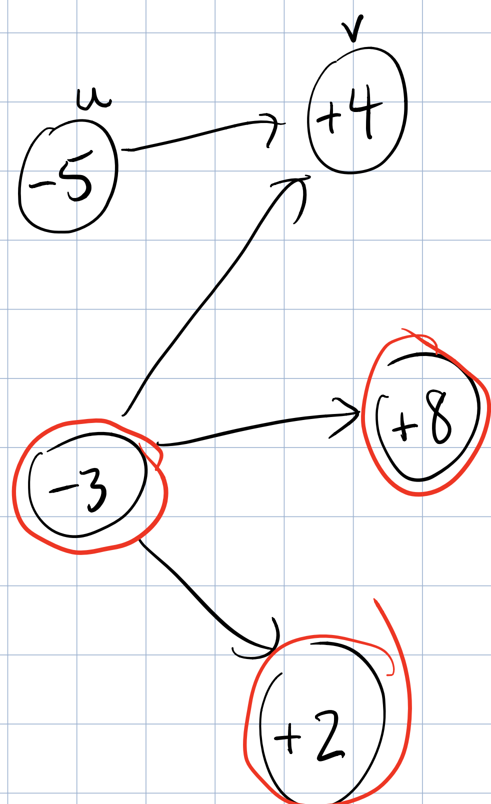
Profit p_i for doing project i.

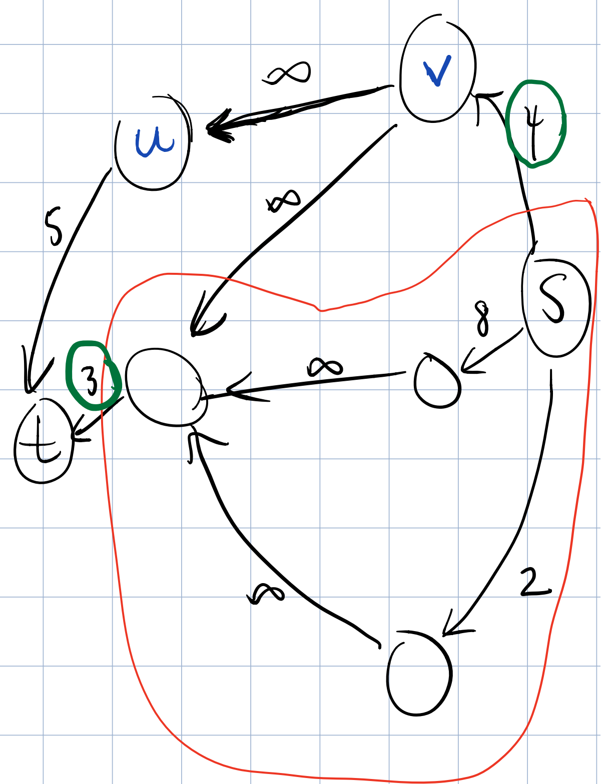
p_i may be positive or negative.

← satisfying dependency constr.

Goal: Choose a feasible subset of projects.
Maximize net profit.

GOAL: $A = \{s\} \cup F$, $B = V - A$ is a finite capacity $\Leftrightarrow$ F is feasible.





One can show (see §7.11) that min-cut (A,B) in network on right has $A = \{s\} \cup \{\text{optimal set of projects}\}$.