7 April 2021 — 3-Dimensional Matching

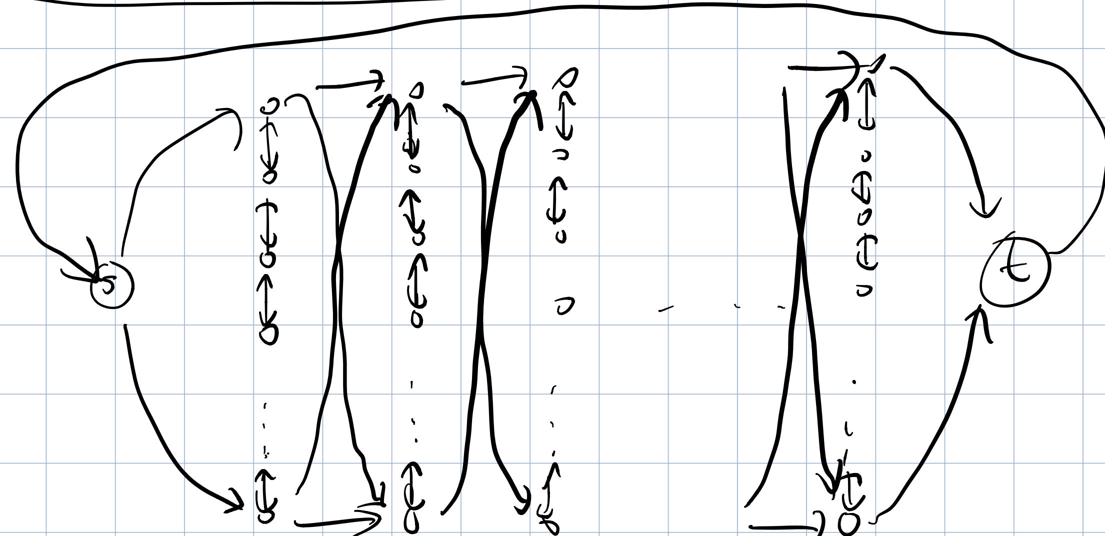
Announcement: Prelim 2 coverage

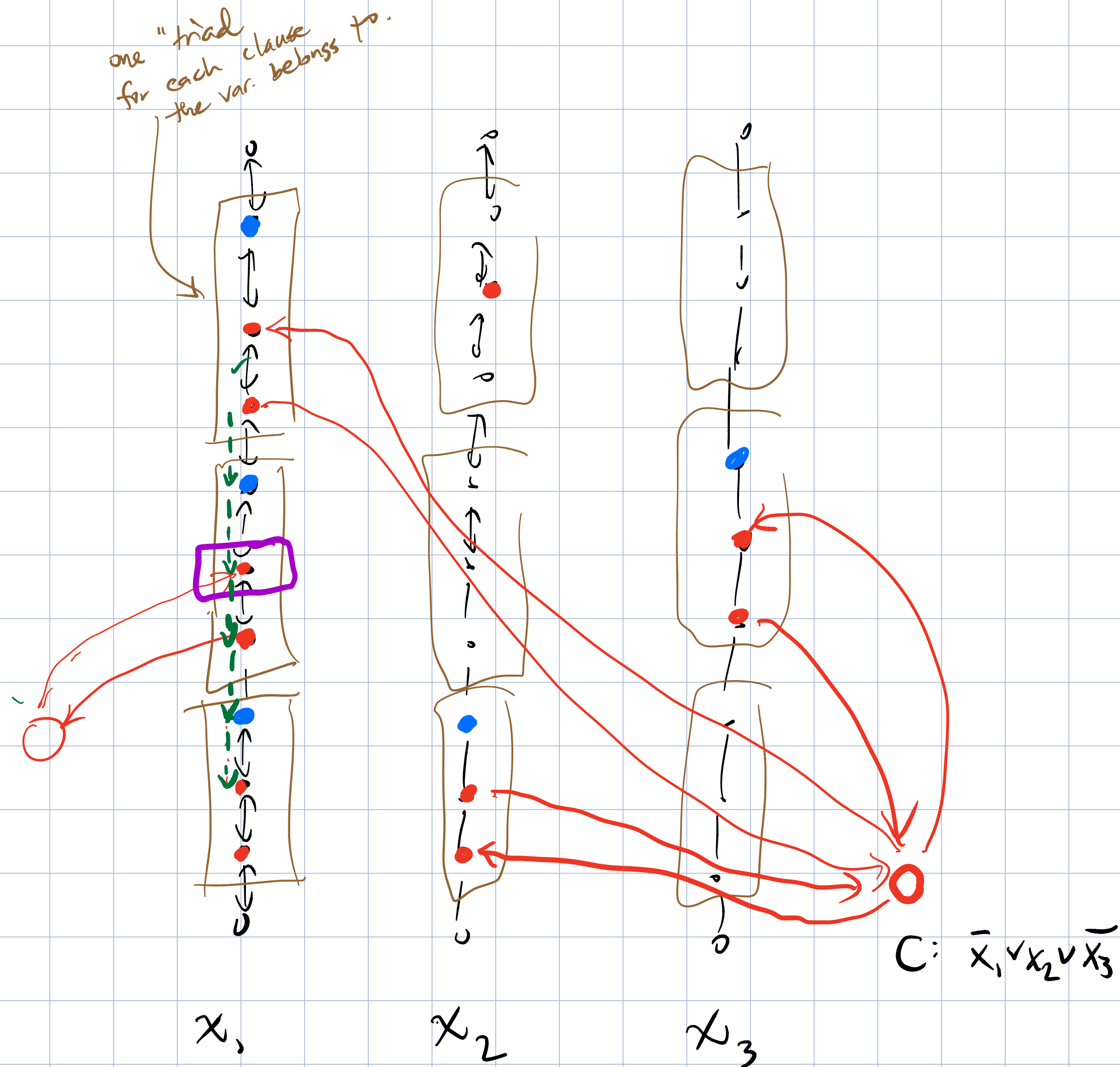
- Chapter 5, 7, 8.1—8.4
- Can use material from this week if you want, but it won't be needed.

Prelim 2 review sessions

- Sat 8–9:30 pm Zoom
- Sun 7–8:30 pm Hollister 314
- Mon 9:05–9:55 in this lecture (Zoom)

See Ed post "Prelim 2 Review Sessions."

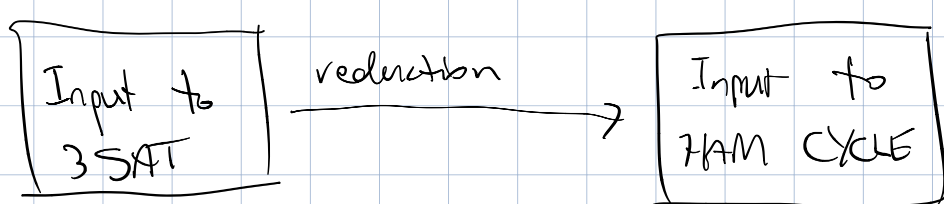




Remains to show: if this dir graph has a Ham cycle, then the 3SAT formula has a satisfying truth assignment.

Lemma: Each blue node has only two neighbors (above and below it). If a Ham cycle goes through any blue node & neighbors in top-to-bottom (resp., B-to-T) order then to must go through **every** blue node of that variable gadget, & neighbors, in the same order.

Using the Lemma we conclude each "variable gadget" is traversed T-to-B or B-to-T. If T-to-B assign true. If B-to-T assign false. This must be a satisfying truth assignment because otherwise if ∃ an unsatisfied clause, there's no way for the Ham cycle to visit its node.



$$\begin{tikzcd} \boxed{\text{Solution to 3SAT}} \arrow[r, shift left, "\text{proof, part 1}"] & \boxed{\text{Solution to HAM CYCLE}} \arrow[l, shift left, "\text{proof, part 2}"] \end{tikzcd}$$

Five steps to solve an NP-Completeness Prob.

1. Show the problem belongs to NP.

   (Generally, describe a poly-time verifier. An algorithm that takes instance of the problem **AND** proposed solution and verifies the solution is valid.)

2. Reduce FROM another NP-Complete problem (e.g. 3SAT) to your problem.

3. Analyze running time of reduction. (Usually 1 sentence.)

4. Proof, part 1: solution of A $\longrightarrow$ solution of B

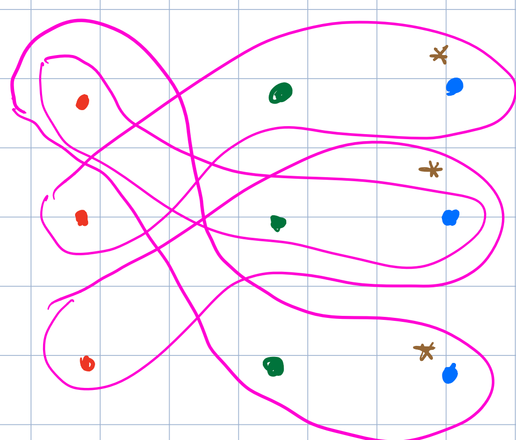
5. Proof, part 2: " " " $\leftarrow$ " " "

Three Dimensional Matching

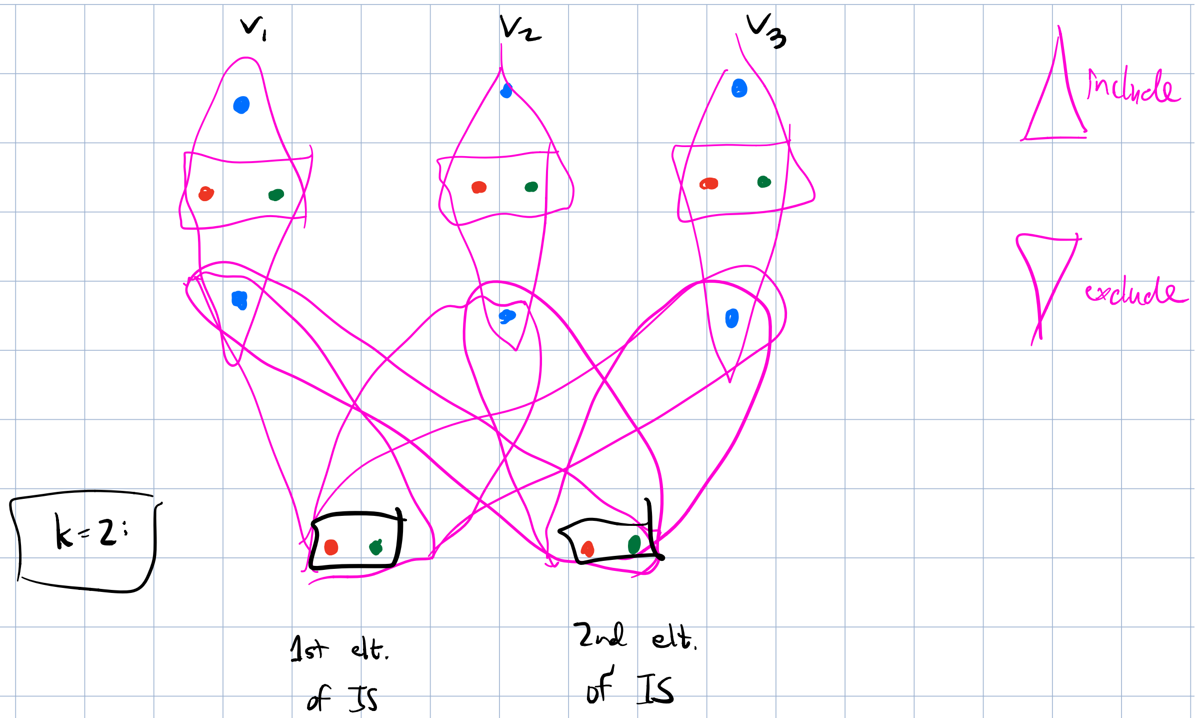
Input: - Three sets R, G, B each with same # of elements, n.

- Collection of m 3-element sets each containing one element from each of R, G, B.

Question: Among the given m 3-elt sets are there n of them which constitute a partition of $R \cup G \cup B$?



To show NP-Hard: reduce from IND SET.



Choosing 3-element sets to cover all red & green nodes in this picture corresponds to choosing a k-element set of vertices.

Q1. How to constrain the k vertices to be non-neighboring?

Q2. There are left over blue nodes. How to cover them?