

## Lecture 29

### Topics

1. PS4 is out, I will add one more problem on IMP design – adding functions with state. We might also consider environments that give names to constants. There is an extra credit problem to explain Blum’s Size Theorem. There will be a PS5 as well.
2. We will study the simple imperative programming language IMP from Winskel’s book *The Formal Semantics of Programming Languages*, MIT Press, 1993. Chapter 2. PS4 includes material from Chapter 10.
3. Functional vs. Imperative languages, “imperative” is for commands, “do this to the state”. We need *assignable* identifiers also called reference variables in the ML family.
4. Winskel gives a syntax, the easy bit, and a *large step* operational semantics (à la Khan) and a *small step* operational semantics (à la Plotkin).
5. We will add *functional procedures* to IMP (to get “FIMP”?). That will be part of PS4. This will be a collective “design” problem.

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### Defining IMP (with some CS6110 flourishes)

#### Abstract syntax

Arithmetic expressions (p.12)

$$aexp ::= n \mid x \mid add(a_1; a_2) \mid sub(a_1; a_2) \mid mult(a_1; a_2)$$

Syntactic conventions

$n, m$	range over $\mathbb{N}$	
$x, y$	range over $Loc$	(locations, variables)
$a$	range over $Aexp$	(arithmetic expressions)
$b$	range over $Bexp$	(Boolean expressions)
$c$	range over $Cmd$	(Commands – Com)

Some “Winskel ways” requiring parenthesis to make the expressions unique – a bit strange to me. Here are his syntactic classes:  $Aexp$ ,  $Bexp$ ,  $Cmd$  (he uses Com for Commands).

I show Algol68 syntax:

**if**  $bexp$  **then**  $c_0$  **else**  $c_1$  **fi**, **while**  $bexp$  **do**  $c$  **od**, and **until**  $bexp$  **do**  $c$  **od**, etc.

**Structured operational semantics** (big step, à la Khan, also called *natural semantics*).

Evaluation requires states. Winskel writes  $\sigma$ , we write  $s$ .  $s : Loc \rightarrow \mathbb{N}$  (recall *Loc* are *locations* in the memory or state).

The rules have the format

$$\langle exp, s \rangle \rightarrow value$$

$$\langle cmd, s \rangle \rightarrow state$$

On page 16 he suggests a *refinement* or top down style of reading the rules. Wirth suggested the same and called them *refinement style* rules. In lecture we will write them this way.

*Evaluation of sums* p.14

*Rule instances* p.15 Note:  $s_0(Init) = 0$

*Derivation tree* p.15

$$\begin{array}{l}
 \vdash \langle (Init) + 5 + (7 + 9), s_0 \rangle \rightarrow \underline{5} + \underline{16} \\
 \quad \vdash \langle (Init + 5), s_0 \rangle \rightarrow \underline{\quad} + 5 \\
 \quad \quad \vdash \langle Init, s_0 \rangle \rightarrow s_0(Init) = 0 \\
 \quad \quad \quad \vdash \langle 5, s_0 \rangle \rightarrow 5 \\
 \quad \quad \quad \quad \vdash \langle (7 + 9), s_0 \rangle \rightarrow \underline{\quad} + \underline{\quad} = 16 \\
 \quad \quad \quad \quad \quad \vdash \langle 7, s_0 \rangle \rightarrow 7 \\
 \quad \quad \quad \quad \quad \quad \vdash \langle 9, s_0 \rangle \rightarrow 9
 \end{array}$$

**Evaluation of Commands** 2.4, p.19

Using  $\langle C, s \rangle \rightarrow s'$

For example:  $\langle X := 5, s \rangle \rightarrow s[5/X]$

List of Commands

skip  $\langle skip, s \rangle \rightarrow s$   
 sequencing  $\langle C_0; C_1, s \rangle \rightarrow$   
 conditionals  $\langle \text{if } b \text{ then } C_0 \text{ else } C_1, s \rangle \rightarrow$   
 while loops (page 20)

$\vdash \langle \text{while } b \text{ do } C \text{ od}, s \rangle \rightarrow s'$   
 $\quad \vdash \langle b, s \rangle \rightarrow true$   
 $\quad \vdash \langle C, s \rangle \rightarrow s''$   
 $\quad \vdash \langle \text{while } b \text{ do } C, s'' \rangle \rightarrow s'$

$\vdash \langle \text{while } b \text{ do } C \text{ od}, s \rangle \rightarrow s$   
 $\quad \vdash \langle b, s \rangle \rightarrow false$