## Problem Set 5

**Reading**: Thompson Chapter 3, Chapter 4: 4.1-4.8. Read the integer square root example posted with the problem set after lecture 37.

## Exercises

1. Define the integers modulo 5 as a CTT quotient type over the integers,  $\mathbb{Z}$ . It will have the form  $\mathbb{Z}//E(x, y)$  where E(x, y) is the equivalence relation defining equality mod 5. Show that plus and times are associative in this type. For small extra credit, comment on the algebraic structure created by this type, e.g. is it a ring, a field?

If you have never studied algebraic structures, for extra credit, compare this method of defining integers mod n to the method used in set theory.

2. Define the type of Turing Machines in CTT and compare it to the definition is standard textbooks on computability theory. We will include in lecture the definition from Hopcroft and Ullman.

Show that the CTT definition can be used to actually execute a Turing machine using the programming language part of constructive type theory.

- 3. Prove that the problem of deciding whether a CTT type is inhabited is undecidable.
- 4. (a) What CTT types can have types as members?
  - (b) Why does Russell's paradox not apply to CTT?
- 5. Read the account of lists as a type, either from Thompson 5.10 or Nuprl rules (posted page 159 of the book).

Prove that every non-empty list of integers has a maximum element.

From the proof of this theorem, extract a program that finds the maximum.

6. Describe an open research problem in the study of programming languages comparable in importance to previous landmark results discussed.