# Assignment 15

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**Definition** (Split Fork)**.** Given morphisms $A \overset{f_1}{\underset{f_2}{\rightrightarrows}} B \xrightarrow{c} C$ of a category, they form a split fork if there exist morphisms $s : C \to B$ and $t : B \to A$ such that the following diagram commutes:

$$\begin{tikzcd}
B \arrow[rrr, "c"] \arrow[dd, "id_B"'] \arrow[dr, "t"] & & & C \arrow[dl, "s"'] \arrow[dd, "id_C"] \\
& A \arrow[r, "f_1"] \arrow[dl, "f_2"] & B \arrow[dr, "c"] & \\
B \arrow[rrr, "c"'] & & & C
\end{tikzcd}$$

**Exercise 1.** Show that, for every split fork $A \overset{f_1}{\underset{f_2}{\rightrightarrows}} B \xrightarrow{c} C$, the morphism $c$ is a coequalizer of $f_1$ and $f_2$.

**Definition** (Absolute (Co)Limit)**.** A (co)limit in a given category $\mathbf{C}$ is called *absolute* if every functor from $\mathbf{C}$ preserves it.

**Exercise 2.** A coequalizer $c : B \to C$ of a pair of morphisms $A \overset{f_1}{\underset{f_2}{\rightrightarrows}} B$ is called split if $A \overset{f_1}{\underset{f_2}{\rightrightarrows}} B \xrightarrow{c} C$ is a split fork. Prove that every split coequalizer is an absolute coequalizer. That is, show that every split coequalizer $A \overset{f_1}{\underset{f_2}{\rightrightarrows}} B \xrightarrow{c} C$ in any category $\mathbf{C}$ has the property that, for any category $\mathbf{D}$ and functor $F : \mathbf{C} \to \mathbf{D}$, the morphism $Fc : FB \to FC$ is a coequalizer of the pair of morphisms $FA \overset{Ff_1}{\underset{Ff_2}{\rightrightarrows}} FB$. You may use the fact that you proved above.

**Exercise 3.** Prove that the multicategory **Prost** is left closed. However, for the existence and uniqueness proofs of $\lambda f$, for the sake of readability only show this for the case where $f$ is binary.