Assignment 2

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Exercise 1. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ that is an isomorphism on $\mathbb{R}$ in **Set** but is *not* an isomorphism on $\langle \mathbb{R}, \leq \rangle$ in $\mathbf{Rel}(2)$.

Exercise 2. Show that the function on objects mapping a set A to the pair of sets $\langle A, A \rangle$, along with the function on morphisms mapping a function f to the pair of functions $\langle f, f \rangle$, constitutes a functor from **Set** to **Set** $\times$ **Set**. (The latter category is defined in 3.3(4f).) This functor is often denoted $\mathbf{Set} \xrightarrow{\Delta} \mathbf{Set} \times \mathbf{Set}$.

Exercise 3. Show that the function on objects mapping the pair of sets $\langle A, B \rangle$ to the set $A \times B$ can be extended into a functor from **Set** $\times$ **Set** to **Set**. This functor is, as an abuse of notation, often denoted $\mathbf{Set} \times \mathbf{Set} \xrightarrow{\times} \mathbf{Set}$.

Exercise 4. The composition of functors $\mathbf{Set} \xrightarrow{\Delta} \mathbf{Set} \times \mathbf{Set} \xrightarrow{\times} \mathbf{Set}$ is equal to what familiar endofunctor on **Set**?

Exercise 5. For each of the following, show whether it is true or false:

a) $\mathbf{Set} \xrightarrow{\Delta} \mathbf{Set} \times \mathbf{Set}$ is faithful.

b) $\mathbf{Set} \xrightarrow{\Delta} \mathbf{Set} \times \mathbf{Set}$ is full.

c) $\mathbf{Set} \xrightarrow{\Delta} \mathbf{Set} \times \mathbf{Set}$ reflects isomorphisms.

d) $\mathbf{Set} \times \mathbf{Set} \xrightarrow{\times} \mathbf{Set}$ is faithful.

e) $\mathbf{Set} \times \mathbf{Set} \xrightarrow{\times} \mathbf{Set}$ is full.

f) $\mathbf{Set} \times \mathbf{Set} \xrightarrow{\times} \mathbf{Set}$ reflects isomorphisms.