Assignment 6

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Exercise 1. Show that **Cat** is a concrete subcategory of some $\mathbf{Alg}(T)$ over **Grph**, meaning there is a functor I from **Cat** to some $\mathbf{Alg}(T)$ that is injective on objects/morphisms and is concrete over **Grph**.

Definition. An object A of a concrete category $\mathbf{A} \xrightarrow{U} \mathbf{B}$ is indiscrete if for every $\mathbf{X}$-morphism $f : UA \to UB$ there exists a (necessarily unique) $\mathbf{A}$-morphism $m : A \to B$ such that $f = Um$.

Exercise 2. Show that a concrete category $\mathbf{A} \xrightarrow{U} \mathbf{X}$ has cofree objects if for every $\mathbf{X}$-object X there exists an indiscrete object in the fibre over X.

Definition. A category is "powered" if for every set I there is a function on objects $I \pitchfork -$ and a bijective mapping from collections of morphisms $\{m_i : B \to C\}_{i \in I}$ to morphisms $\pitchfork_{i \in I} m_i : B \to I \pitchfork C$ such that for every morphism $f : A \to B$ the equality $\pitchfork_{i \in I} (f \,; m_i) = f \,; \pitchfork_{i \in I} m_i$ holds. Because $\pitchfork$ is bijective (on morphisms), for a morphism $m : B \to I \pitchfork C$ let $\{\pitchfork_i^{-1} m : B \to C\}_{i \in I}$ denote the collection of morphisms that $\pitchfork$ maps to m.

Exercise 3. Show that every category with small (i.e. set-indexed) products is powered.