Practice 9

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Exercise 1. An idempotent semigroup is a semigroup $\langle A, + \rangle$ satisfying the property $\forall a \in A. a + a = a$. For example, min and max on $\mathbb{N}$ are both idempotent semigroups. Show that the full subcategory of $\mathbf{Sgr}$ containing precisely the idempotent semigroups is *co*reflective.

Exercise 2. Show that a transposition is precisely an isomorphism between the sources $\mathbf{C} \leftarrow (L{\downarrow}\mathbf{D}) \rightarrow \mathbf{D}$ and $\mathbf{C} \leftarrow (\mathbf{C}{\downarrow}R) \rightarrow \mathbf{D}$ in $\mathbf{C}$. (See here for a refresher on notation and for historical insight.)

Exercise 3. Using only 2-cell diagrams, prove that adjunctions can be sequentially composed (similarly to how transpositions can be sequentially composed).

Exercise 4. Viewing $\mathbf{Rel}(1)$ as a concrete category over $\mathbf{Set}$, there is a functor (denoted $\pi^*_{X,Y}$) from the fibre over a given set X to the fibre over the set $X \times Y$ given by $\phi \subseteq X \mapsto \{\langle x, y \rangle \mid x \in \phi \wedge y \in Y\} \subseteq X \times Y$. Give the left adjoint and give the right adjoint to this functor. (Hint: each corresponds to a common construct in logic.)