# Functors

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**Example.** The function mapping $\langle A, e, *, ^{-1} \rangle$ to $\langle A, e, * \rangle$ extends to a full and faithful functor from **Grp** to **Mon** (that consequently reflects isomorphisms).

**Example.** The function mapping a set $A$ to the monoid $\langle \mathbb{L}A, [\,], + \!\!+ \rangle$ extends to a faithful functor from **Set** to **Mon**, with the function $f : A \to B$ mapping to the monoid homomorphism $\mathrm{map}(f) : \langle \mathbb{L}A, [\,], + \!\!+ \rangle \to \langle \mathbb{L}B, [\,], + \!\!+ \rangle$.

**Example.** The function mapping a set $A$ to itself and the function $f : A \to B$ to the relation $\{\langle a, f(a) \rangle \mid a \in A\} \subseteq A \times B$ extends to a faithful functor from **Set** to **Rel** that reflects isomorphisms.

**Example.** The function mapping a set $A$ to its power set $\mathbb{P}A$ and the relation $R \subseteq A \times B$ to the function $\lambda x : \mathbb{P}A.\ \{b \mid a \in x, a\ R\ b\}$ extends to a faithful functor from **Rel** to **Set** that reflects isomorphisms.

**Example.** The function mapping $n \in \mathbb{N}$ to the set $\mathbb{B}^n$ and mapping a circuit to the function on Booleans that it implements extends to a full functor from **Circ** to **Set**.

**Example.** The function mapping a circuit to its graph of gates and wires does *not* extend to a functor from **Circ** to **Graph** because circuits are *morphisms* in the former and graphs are *objects* in the latter.

**Example.** The function mapping a deterministic automaton to the graph whose vertices are the states of the automaton and whose edges are the transitions of the automaton labeled with the appropratiate character extends to a faithful functor from $\Sigma$-**Seq** to $\Sigma$-**Graph**.

**Example.** The function mapping a deterministic automaton to the set of strings accepted by the automaton extends to a functor from $\Sigma$-**Seq** to $\Sigma$-**Lang**.

**Example.** The function mapping $n \in \mathbb{N}$ to the set $\mathbb{R}^n$ and mapping a matrix to the linear function it specifies extends to a faithful functor from **Mat** to **Set** that reflects isomorphisms (and a full and faithful functor to **Vec**, for those familiar with vector spaces).