Isomorphisms

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Example. The monoid $\langle \mathbb{R}, 0, + \rangle$ is isomorphic in **Mon** to the monoid $\langle \mathbb{R}^{>}, 1, * \rangle$ as evidenced by $\lambda x.\ e^x$ and $\lambda x.\ \ln(x)$.

Example. The above also extends to an isomorphism in **Grp** between $\langle \mathbb{R}, 0, +, \lambda x.\ -x \rangle$ and $\langle \mathbb{R}^{>}, 1, *, \lambda x.\ \frac{1}{x} \rangle$.

Example. A relation $R : A \to B$ in **Rel** is an isomorphism iff both $\forall a \in A. \exists! b \in B.\ a\ R\ b$ and $\forall b \in B. \exists! a \in A.\ a\ R\ b$ hold.

Example. Two graphs are isomorphic if conceptually one is simply a "renaming" of the vertices and edges of the other.

Example. The only isomorphisms in **Circ** are the identity morphisms.

Example. $\neg : \mathbb{B} \to \mathbb{B}$ is its own inverse in **Set**.

Example. $\neg : \langle \mathbb{B}, \mathbb{t}, \wedge \rangle \to \langle \mathbb{B}, \mathbb{f}, \vee \rangle$ is the inverse of $\neg : \langle \mathbb{B}, \mathbb{f}, \vee \rangle \to \langle \mathbb{B}, \mathbb{t}, \wedge \rangle$ in **Mon**.

Example. Negation is an endomorphism on $\langle \mathbb{Z}, 0, + \rangle$ that is its own inverse in **Mon**.

Example. Negation serves as an isomorphism in $\mathbf{Rel}(2)$ between $\langle \mathbb{R}, \leq \rangle$ and $\langle \mathbb{R}, \geq \rangle$ in both directions.

Example. Negation does *not* serve as an isomorphism in $\mathbf{Rel}(2)$ between $\langle \mathbb{R}, \leq \rangle$ and $\langle \mathbb{R}, > \rangle$.

Example. When interpreting a group as a single-object category, *every* morphism (i.e. element of the group) is an isomorphism.